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OF THE
ROYAL ASTRONOMICAL SOCIETY.

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No. 8

Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Frederick Hurn Constable, M.A., Ph.D., D.Sc., St. John's College, Cambridge ;

Theodore Dunham, Jr., Ph.D., M.D., Mount Wilson Observatory, Pasadena, Cal., U.S.A. ;

Alfred L. Loomis, Loomis Laboratory, Tuxedo Park, New York City, U.S.A. ; and

Andrew Cecil Telfer, M.A., Dinham Hall, Ludlow, Salop.

One hundred and four presents were announced as having been received since the last meeting, including, amongst others :—

Colombo Observatory Bulletins, vol. i., pts. 1, 2.

H. Purey-Cust, Azimuth by Logs.

Uccle, Observatoire Royal de Belgique, Annales, Tome 2, fasc. 2.

Carte photographique du

Ciel, zones + 33°, + 35°, + 37°, + 39° (30 charts).

The Pleiades. (George Darwin Lecture, delivered by Professor E. Hertzsprung on 1929 May 10.)

Mr. President,—First of all I want to thank you for the honour shown me by asking me to be the third in the rank of those who are giving the George Darwin Lecture.

I will, however, leave it to others to take up problems more related to the fundamental work linked with the name of this lectureship. The subject about which I am going to speak during the next hour will be taken from quite another field of astronomy.

I have announced my lecture to be about the Pleiades. I ask your excuse for this too general title. I am not going to cover the whole field indicated by the name of that famous group, but shall confine

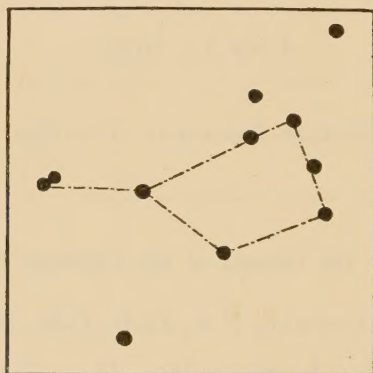


FIG. 1.—Pleiades according to Maestling, 1579.

myself to some more recent, partly unpublished, investigations concerning this subject.

The Pleiades form perhaps the most conspicuous of all stellar clusters. To a normal naked eye about 6 stars are visible, and to an unusually sharp eye 11 stars within an area of 3 or 4 square degrees. This number is shown on the map of the Pleiades made by Maestling in 1579, now just 350 years ago, which was before the invention of the telescope (fig. 1). The catalogue which we have made in Leiden from a plate taken at Algiers contains 2616 stars. I do not dare to make the extrapolation for the next 350 years to come.

The plausible suggestion that these brighter stars belong physically together is, as it has long been known, supported by the fact that they have practically identical proper motions and very similar spectra.

The important question now naturally arises, how the fainter stars projected on the sky within the group behave in this respect. How many physical members are to be found among these fainter stars and how do their spectra or colours eventually vary with their apparent magnitudes?

After application of the most modern correction to the proper motions found with the meridian instrument, the mean proper motion of the 12 brightest stars in the Pleiades is $+ 0''.020$ in $\alpha \cos \delta$ and $- 0''.044$ in δ , or a total proper motion of $0''.048$ in the direction 156° . The direction from the apex of the sun's motion is 147° .

In order to penetrate to stars fainter than those reached by the meridian circle, the photographic method suggests itself. This method requires comparison between old and new plates taken several years apart. This requirement is in the case of the Pleiades perhaps better fulfilled than any other at present. There is some reason for this fact.

The apparent diameter of the Pleiades is about 2 degrees. That is to say, they just cover a normal *carte du ciel* plate. Therefore the Pleiades, which show both bright and faint stars all over the plate, have formed a favoured test-object for refractors of the *carte du ciel* type. As soon as such a refractor had been erected, the Pleiades were among the first objects to be photographed. And then the nebulae present in the group are just bright enough to show on plates taken with the aperture-ratio of the *carte du ciel* refractor, 1 : 10, with an exposure-time of a few hours. This circumstance may also have some bearing on the fact, that at different observatories there are old, rather-long-exposed plates of the Pleiades present, which, when repeated now, will enable us to determine fairly accurate values of the relative motions of the fainter stars in the group.

But in order to reach the best accuracy obtainable in our days it is necessary to gather all the available old material together and to get new plates taken through the glass as far as possible with the same instruments. This requires extended international collaboration, and I am glad to say that the Leiden Observatory has been helped in the most generous way by a number of observatories in different countries providing us with the necessary material. It is only thanks to these valuable contributions from other observatories that this investigation could be carried through. So far we have obtained, and to a great extent already worked up in Leiden, plates from the following observatories, which I shall mention in alphabetical order. They are: Algiers, Bonn, Greenwich, Helsingfors, Leiden, Paris, Potsdam, Pulkovo, and Tashkent. Nearly all the plates have been taken with refractors of the *carte du ciel* type—only those of Bonn and Leiden with a focal length one and a half times longer. We hope to get still more plates.

The well-known method of differential measures has been used throughout. The old plate and the new one taken through the glass are placed film against film and shifted so much against each other that each star appears as a measurable double.

One of the greatest difficulties in comparing old and new photographic plates for accurate determinations of relative proper motions is caused by the presence of a variable magnitude equation in the position of the stars. Nowadays we may practically eliminate the magnitude equation by the aid of a coarse grating placed in front of the objective, thus producing faint diffraction images of the brighter stars. But the old plates have not been taken in this way.

There is, however, another possibility. Suppose that the relative proper motions have been found with sufficient accuracy to discriminate with fair certainty between the physical members of the group and other stars, and that there is a sufficient number of such physical members. We then only need to assume that there are no systematic differences between the proper motions of bright and faint physical members of the group before using these stars for the elimination of the magnitude equation. That is what has actually been done at Leiden. The proper motions of the physical members of the group have been used as zero-points in all the calculations.

The measures of each pair of plates have been treated according to least squares, using linear formulæ only. When once final mean values, derived from all the material available, have been obtained, it is the intention to make a new least squares solution, if necessary including terms of the second order.

The results I want to present to you this evening are only based on that part of the material which has so far been measured and reduced.

In order to give you an idea of the accuracy obtained by the gathering together of so many pairs of plates, I have chosen two examples, presented in Table I. The first column gives the designation of the pair of plates measured; the next two columns give the components of the proper motion in $\alpha \cos \delta$ and δ in seconds of arc in 1000 years relative to the physical members of the group; the last column gives the weight in such units that a weight of 10 corresponds to a mean error of ± 10 in the proper motion as given. The assigned weights have been controlled and seem to be trustworthy.

TABLE I.

Gaultier 13. ($13^m.2$)				Gaultier 57. ($13^m.0$)			
Pair.	Rel. p.m. in secs. of arc per 1000 years.		Relative Weight.	Pair.	Rel. p.m. in secs. of arc per 1000 years.		Relative Weight.
	$\alpha \cos \delta$.	δ .			$\alpha \cos \delta$.	δ .	
B	- 3	+ 11	14	B	- 7	+ 20	14
C	+ 7	+ 21	5	D	- 2	+ 8	14
D	- 3	+ 12	14	E	- 5	+ 6	30
E	- 7	+ 14	28	F	- 6	+ 10	6
F	- 16	+ 9	5	G	+ 8	+ 19	14
G	- 1	+ 5	14	H	- 5	+ 26	15
I	- 13	+ 20	17	I	- 15	+ 2	17
L	- 16	+ 5	17	L	- 10	+ 5	18
M	- 14	+ 16	20	M	- 10	0	22
O	+ 3	+ 12	32	O	+ 3	+ 10	32
P	- 8	+ 7	14	P	+ 3	+ 5	16
T	- 11	+ 8	50	T	- 3	+ 6	50
Mean	- 7	+ 11	230	Mean	- 4	+ 9	248

As you will see, the two stars have both proper motions very nearly equal to that of the group of the Pleiades, but not quite so. The pre-

valence of the positive sign in the declination-component makes it very probable that the deviations from the mean motion of the group-stars are real.

A total weight of 250 corresponds to a mean error of $\pm 2''$ in 1000 years.

After this introduction I am going to show you some of the results in graphical representation (see fig. 2). First we will consider stars of each magnitude class separately. I will begin with the very brightest stars and go on in steps of one magnitude each.

Between the photographic magnitudes 2^m and 3^m there is only one star, the brightest of the group, Alcyone. Its proper motion is represented on the first diagram by a dot, the cross indicating the zero-point of the proper motions as derived from meridian observations. One square division corresponds to a proper motion of $10''$ in 1000 years.

The next diagram shows in the same way the proper motions of the 3 stars Atlas, Electra, and Maia, between 3^m and 4^m . And so forth, 4^m - 5^m , 5^m - 6^m . So far the proper motions depend on meridian observations, as these bright stars are vastly over-exposed on the plates measured. The greater scattering on the diagram for the 4 stars between 5^m and 6^m therefore does not mean anything serious. The 4 stars are almost certainly physical members of the Pleiades.

On the next diagram, 6^m to 7^m , there are 5 stars again all showing the same proper motion. We go further, to the stars between 7^m and 8^m , and here the first 3 stars, not showing the proper motion of the group, appear: two of them fall outside the diagram.

On the next diagram, 8^m to 9^m , there is also only one star with deviating proper motion, all the 16 others moving with the group.

On the next diagram, 9^m to 10^m , the picture begins to change. There are 5 erratic stars and 13 more or less probable members of the group.

To continue. On the diagram for the stars between 10^m and 11^m there are a dozen stars of peculiar proper motion and 15 with a proper

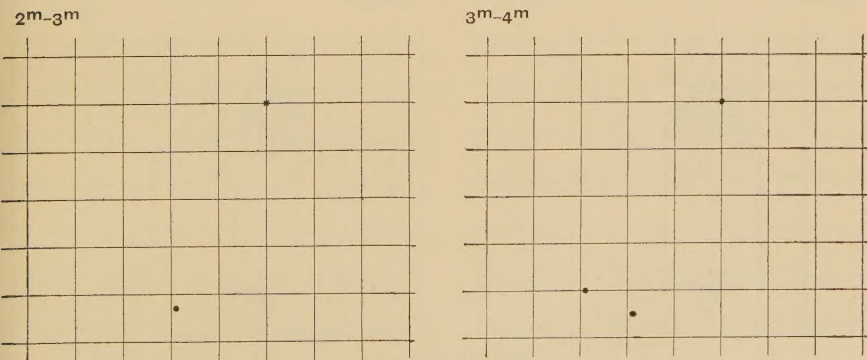


FIG. 2.—Abscissae = δ . Ordinates = $\alpha \cos \delta$.

(The cross indicating zero-point is at the junction of the 2nd horizontal line and the 3rd vertical line from the right in each diagram.)

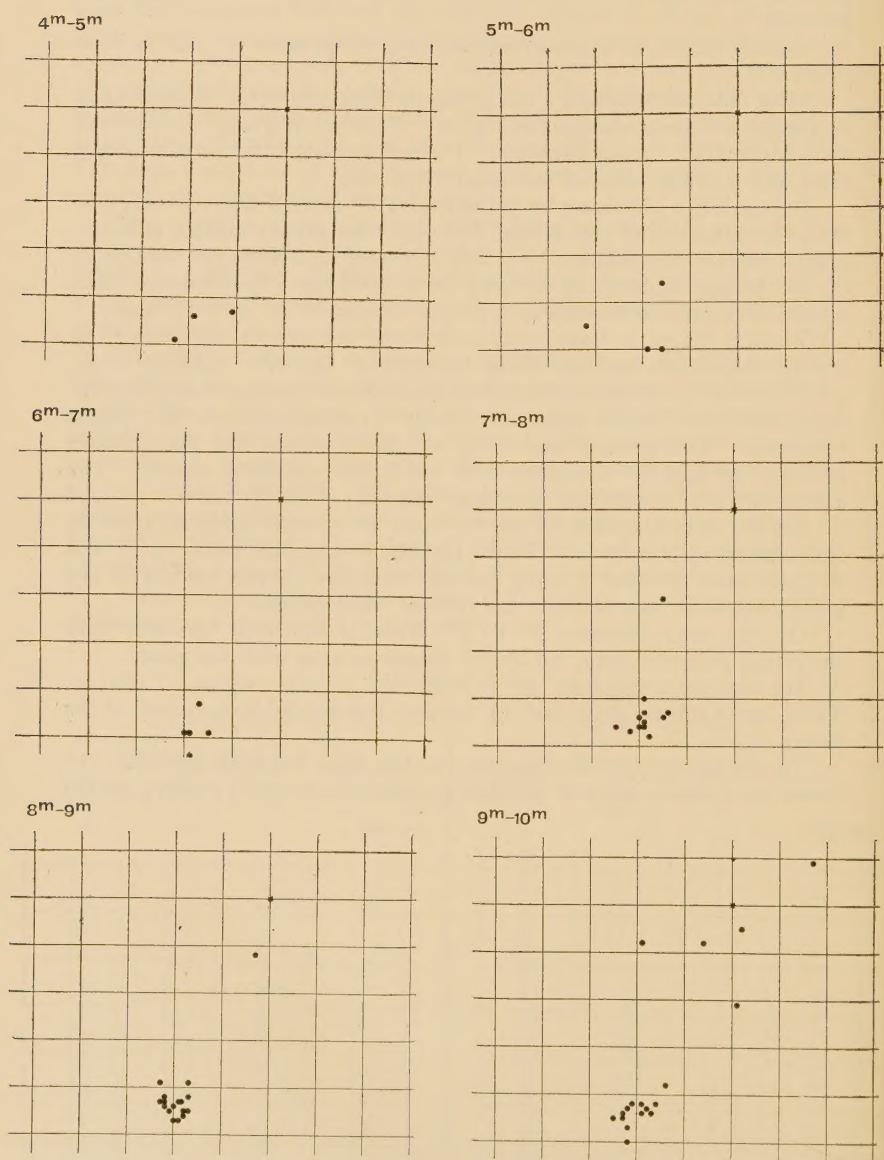


FIG. 2 (continued).

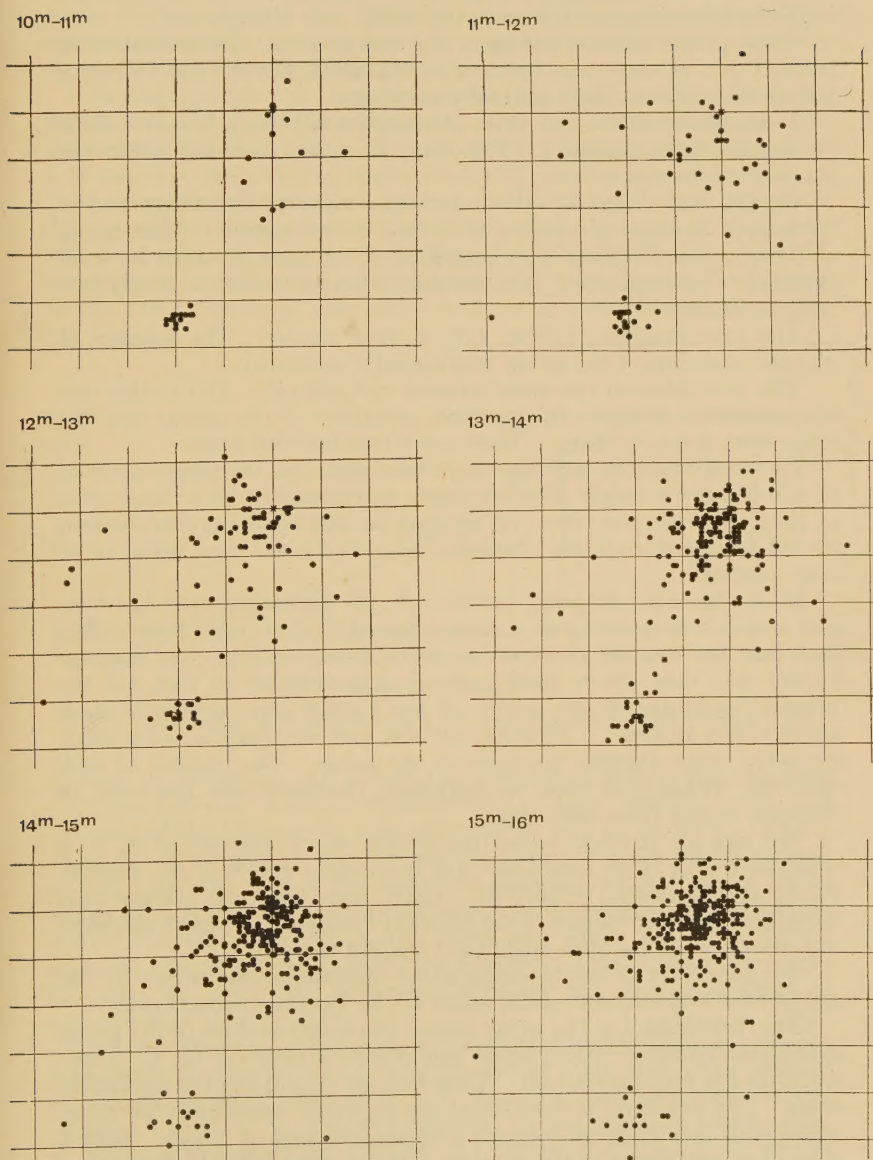


FIG. 2 (continued).

motion very nearly equal to that of the physical Pleiades. In other words: For this magnitude class 10^m to 11^m , about half of the stars in the field investigated belong to the group, half of them not.

These proper motions belong to the best derived. Their mean error is about $\pm 2''$ in 1000 years for each co-ordinate. For the now following fainter stars the accuracy gradually decreases.

I should say that a few stars of exceptionally large proper motion lie outside these diagrams. This fact, however, does not affect the general conclusions.

On the next diagram, 11^m to 12^m , the stars of the background or foreground increase in number in a way to be expected. The group of the physical Pleiades, here consisting of 17 stars, is still very conspicuous. The separation of these stars from the others is nearly free from ambiguity.

The next diagram, 12^m to 13^m , is very similar. The number of physical members of the group is practically unaltered.

This still holds for the stars between 13^m and 14^m . But in this case the separation between the physical members of the group and the other stars is not so sharp. There are a few doubtful cases.

On these diagrams only stars have been included, the proper motions of which have a weight of 60 or more, corresponding to a mean error of about $\pm 4''$ in 1000 years. If we want to look for the proper motions of still fainter stars in the Pleiades, we have for the time being to be more modest.

So for the next diagram, 14^m to 15^m , the limiting weight has been put at 40, corresponding to a mean error of $\pm 5''$ in 1000 years. The fact that the number of stars in general increases from one diagram to the next does not in itself make it more difficult to pick out the physical members of the group, as the fainter stars generally show small proper motions. Between 14^m and 15^m we count a dozen stars, probably being physical members of the group. The number of such members within each class of magnitude therefore does not seem to decrease among these faint stars.

We now go down to a limiting weight of 18, corresponding to a mean error of $\pm 7\frac{1}{2}''$ in 1000 years, and then take a glance at the diagram for the stars between 15^m and 16^m . Again, there are about a dozen stars showing the proper motion of the physical Pleiades. We have therefore not even here at the 16th magnitude reached the end.

We hope next to be able to go about one magnitude further by the aid of very-long-exposed plates taken at the Tashkent Observatory.

The distribution of the so far known physical members of the group among the different photographic magnitudes is shown in fig. 3. These numbers are very provisional. They may be incomplete for the fainter stars. The numbers, as they stand, are tolerably satisfied by an error-curve with maximum at 13^m and a mean deviation of $\pm 4\frac{1}{2}^m$. Such a curve has been drawn in the diagram. It is possible that the maximum frequency will be found to be at a still fainter magnitude, when more material for fainter stars is available. But it may already be said, that the part of the frequency-curve known at present, considered as

a part of an error-curve, indicates a mean deviation of ± 4 or ± 5 magnitudes. This is rather much, remembering that the luminosity-curve of Kapteyn and van Rhijn for stars in the surroundings of our sun corresponds in the same way to a mean deviation of $\pm 2^m.5$ visually or about $\pm 3^m$ photographically.

Another question is, how the known physical Pleiades are distributed within the field investigated. This is shown in fig. 4. The diameter of the circle is 2° . The most striking feature of the diagram is, I think, that the physical members of the Pleiades appear to be only little concentrated towards the centre within the circle of 2° diameter. This seems to indicate that so far only the central portion of the group has been investigated. In fact, Trumpler* has looked for physical members of the Pleiades up to a distance of 3° from Alcyone, and found in the outer regions of that field a number of stars sharing the proper

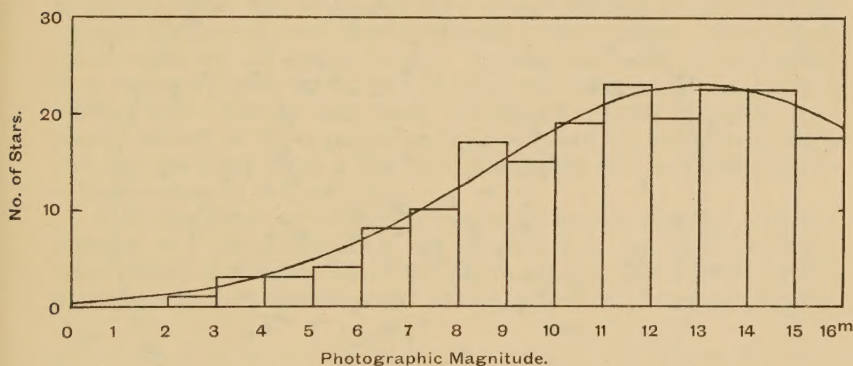


FIG. 3.—Frequency of Physical Pleiades of Different Photographic Magnitudes.

motion of the group. Of all his 134 probable members brighter than 12^m , one half part is found within a circle of $2400''$ semi-diameter. In the case of the Hyades the semi-diameter of the corresponding circle is $10,000''$. If we assume that the real diameter of the two groups defined in this primitive way is the same, the parallax of the Pleiades works out at $0''.0065$. It may be remembered that the parallax of the globular cluster Messier 3 derived in the same way works out at $0''.0001$,† agreeing with the value assumed by Shapley. This method—if it may be called one—has the advantage of being free from the effect of absorption of light in space.

It is pretty certain that there are no stars brighter than, say, the 6th magnitude physically belonging to the group other than those already recognized as such. But it is not necessary, and according to von Zeipel not even to be expected, that the fainter members should show as strong a concentration towards the centre of the group as do the several bright stars.

We have seen that it is now possible to make a fair separation between

* *Lick Bull.*, 333, 1921.

† *A.N.*, 4952, 1918.

the physical members of the group and other stars. The next question is then, whether the proper motions of the physical members are accurate enough to show the small internal differences which we for several reasons are bound to expect.

I will mention a few causes for such real or apparent internal motions.

1. If the Pleiades all move with exactly the same speed and in the

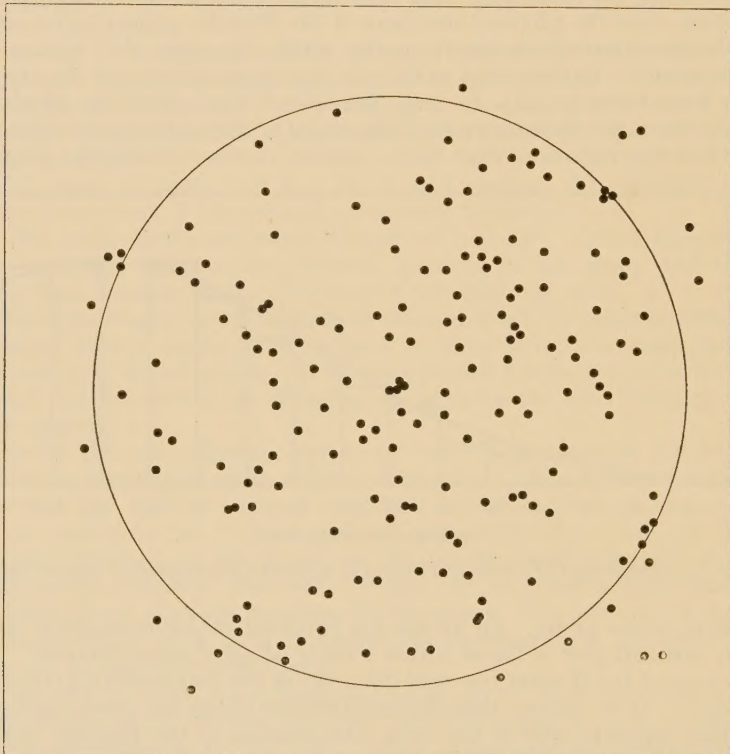


FIG. 4.—Distribution of Physical Pleiades. (Diameter of circle, 2° .)

same direction, the members of the group nearer to us will show a greater apparent proper motion than those farther away, just like soldiers in the ranks of a passing regiment.

Suppose the Pleiades to be a globular cluster with an apparent diameter of 2° . Then the distances from us to the nearest stars and those farthest away will be about 4 per cent. different. As the apparent proper motion of the Pleiades is $0''.048$ this corresponds to a difference of $0''.002$. This amount is about equal to the mean error of our best proper motions. We are therefore not so very far from a sensible influence of this circumstance. The effect as seen on the diagram will be an elongation in the group of points representing the proper motion

of the physical members. This elongation will be in the direction of the proper motion of the system, such as shown on the diagram for the stars between 13^m and 14^m . That the elongation appears here for these faint stars, while it is absent on the diagrams for the brighter stars, may be caused by greater scattering in space of the stars of relatively small mass. But I do not believe that the elongation shown here is real. The diagram only illustrates what I wish to say.

2. Our sun is going away from the Pleiades with a speed of about 8 km./sec. As a consequence of this the group will apparently shrink. If also the distance of the group were known, the amount of this apparent shrinking could be easily calculated. In the case of the Hyades the linear shrinking is one-millionth yearly. Taking the parallax of the Pleiades to be half that of the Hyades, then the proportion between the radial velocities being about 1 to 5, the linear shrinkage of the Pleiades should be about one in ten millions yearly; that is to say, that an angular distance of $7200''$ would diminish $0''.0009$ yearly. This is just below what we can verify to-day. Besides, the effect is eliminated from the group stars by the way in which the differential measures are reduced here. On the other hand, the effect is by this procedure artificially thrown on the apparent proper motions of the background stars and may be found there. The two co-ordinates give two independent determinations of the shrinking.

3. So far, about 150 physical Pleiades are known within a projected distance of 1° from Alcyone. Say that there is 200 times the sun's mass within a distance of 1° or about 300,000 astronomical units. A particle at this distance would have a period of 10,000,000 years in

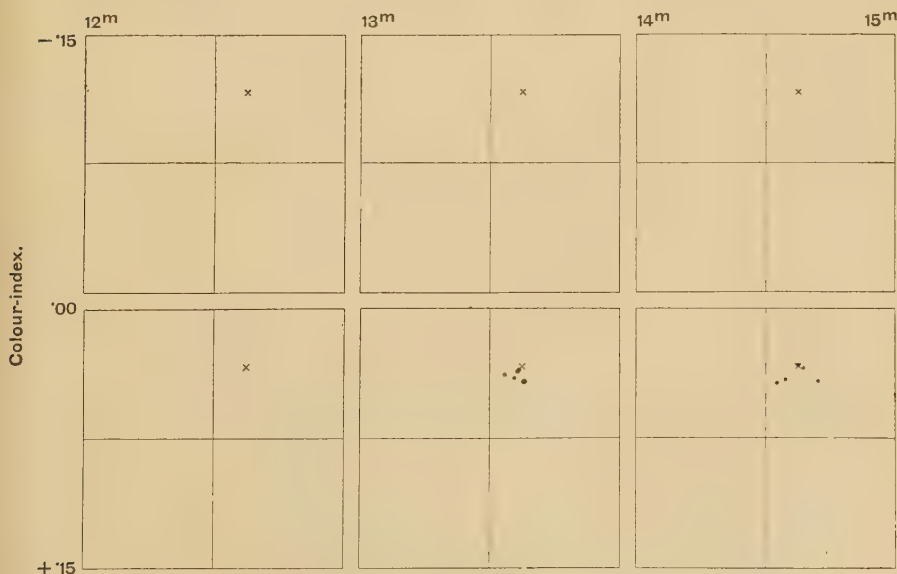


FIG. 5.



FIG. 5 (continued).

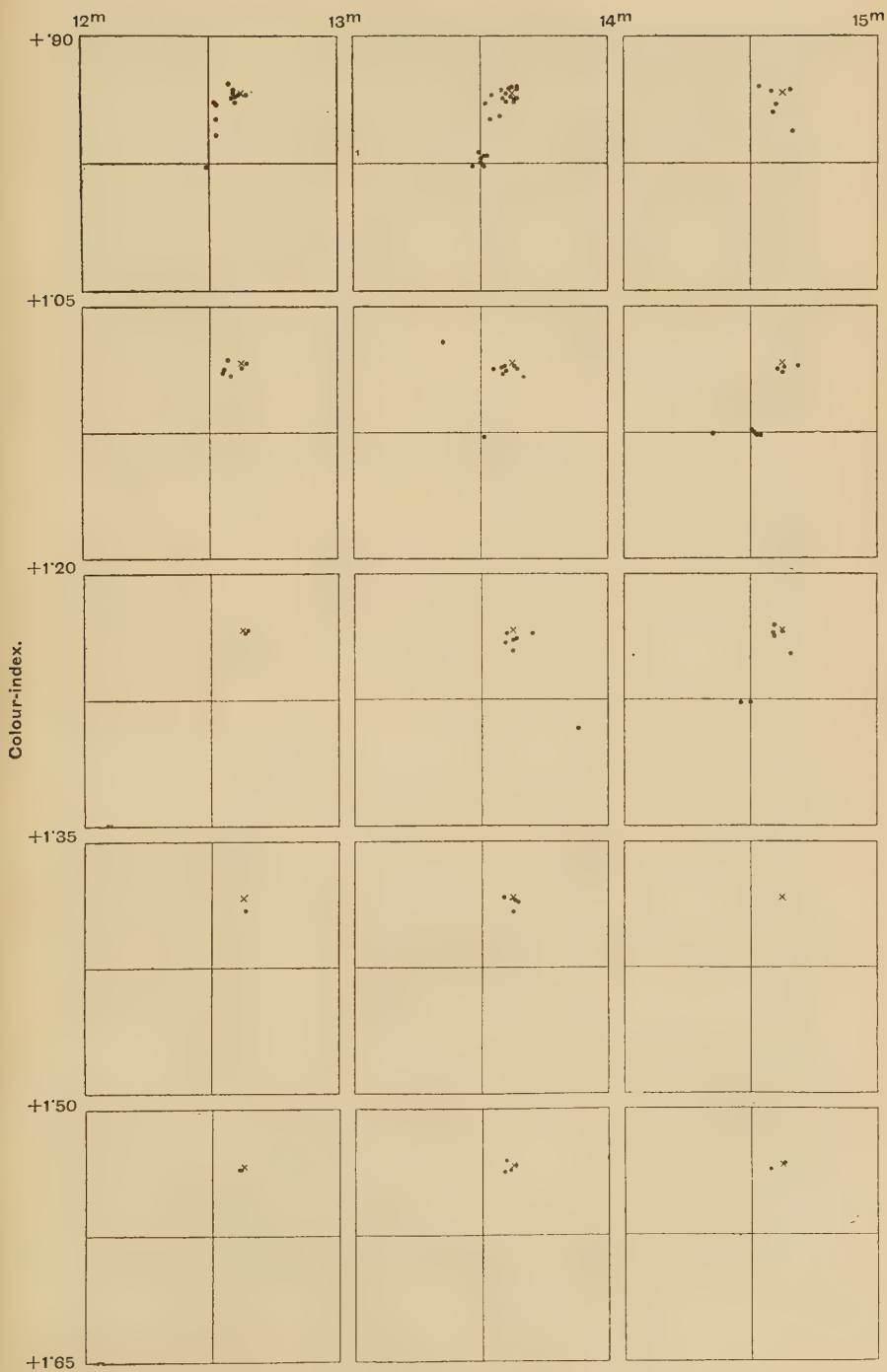


FIG. 5 (continued).

a circular orbit, or an apparent orbital motion of $0''.0023$ yearly. This again is of the same order of magnitude as the mean error of the best proper motions determined.

In order to see if any internal motions are shown by the results so far obtained, I have picked out the 33 physical members of the group, the proper motions of which have the highest weight. The mean error of the annual proper motion in each co-ordinate for one of these stars is $\pm 0''.0019$ according to the assigned weights, while from an intercomparison of the same proper motions, supposing them all to be really equal, a mean error of $\pm 0''.0019$ is derived, or exactly the same as that corresponding to the assigned weights. This means that the internal motions in the group are not yet conspicuous—unless the assigned weights of the proper motions are too small. But I have no reason to believe that this should be the case. The general rule is, that assigned weights are too great.

At any rate it is not at all likely that the total mass of the system of the Pleiades is many times greater than that of the stars already known to be members of the group. We therefore cannot expect to find among the still fainter physical Pleiades a very great number of Eddington's white dwarfs.

So far we have not considered the colours of the stars. We are going to do that now.

In the Pleiades, fairly accurate colour equivalents are known down to about 14^m photographically, mainly depending on the measurement of the effective wave-lengths.* All the colour equivalents have been converted into the scale of colour-indices, *i.e.* the difference between photographic and visual magnitudes.

As it has proved desirable to divide the stars into rather small groups according to colour, the relatively numerous faint stars are best fitted for a statistical treatment. I shall then consider the stars of the magnitude groups 12^m to 13^m , 13^m to 14^m , and 14^m to 15^m separately, and further divide each of these groups into stars of different colour, *viz.* colour-indices — $\cdot 15$ to $\cdot 00$, $\cdot 00$ to $\cdot 15$, $\cdot 15$ to $\cdot 30$, etc., up to the reddest stars here found, the latest group being $1\cdot 50$ to $1\cdot 65$. For each of the $3 \times 12 = 36$ groups the proper motion diagram (fig. 5) has been made in the same way as before, the limiting weight being 60, corresponding to a mean error of about $\pm 0''.004$ yearly.

These diagrams are on a smaller scale, and in this case not a single star has been left out because of exceptionally large proper motion. It is rather fortunate that this has been possible, as Barnard's proper motion star, on the scale of the diagrams as projected, would be placed in another room of this building.

The centre of the diagrams corresponds to the proper motion of the physical members of the Pleiades. The ordinary zero-point of the proper motions is marked on each diagram with a cross. The side in each of the small squares corresponds to a proper motion of $80''$ in a thousand years.

* *Mém. de l'Acad. Roy. des Sciences et des Lettres de Danemark*, Sect. des Sc., 8^{me} sér. t. iv, No. 4, 1923.

It is most surprising that the first row, where the white stars with colour-index from -0.15 to 0.00 ought to be, is empty.

There may be some uncertainty as to the zero-point of the colour-indices adopted, but considerable care has been taken to eliminate the magnitude-equation of the colours, and it is as certain as I can make it that there are no faint stars in this region as white as the brightest stars in the Pleiades.

Looking more closely at the 5 of the first 9 diagrams which contain stars, it is seen that all the proper motions shown are small, which is to be expected of faint stars with colour-indices between 0.00 and 0.30 . Not a single physical member of the Pleiades is present on these diagrams.

The next 9 diagrams, for colour-index between 0.30 and 0.75 , show stars of somewhat lower effective temperature, about like our sun. The difference in spread between brighter and fainter stars of the same colour is well marked. The increase in the spread compared with the whiter stars of the first 9 diagrams is evident.

It is of special interest to note the striking difference between the two following diagrams. On the diagram 12^m to 13^m , colour-index 0.45 to 0.60 , 9 stars of various, rather large proper motions are shown, one of which may be a physical member of the Pleiades. On the diagram below, of the slightly yellower stars of the same brightness, colour-index 0.60 to 0.75 , there are 9 physical Pleiades and only 2 erratic stars.

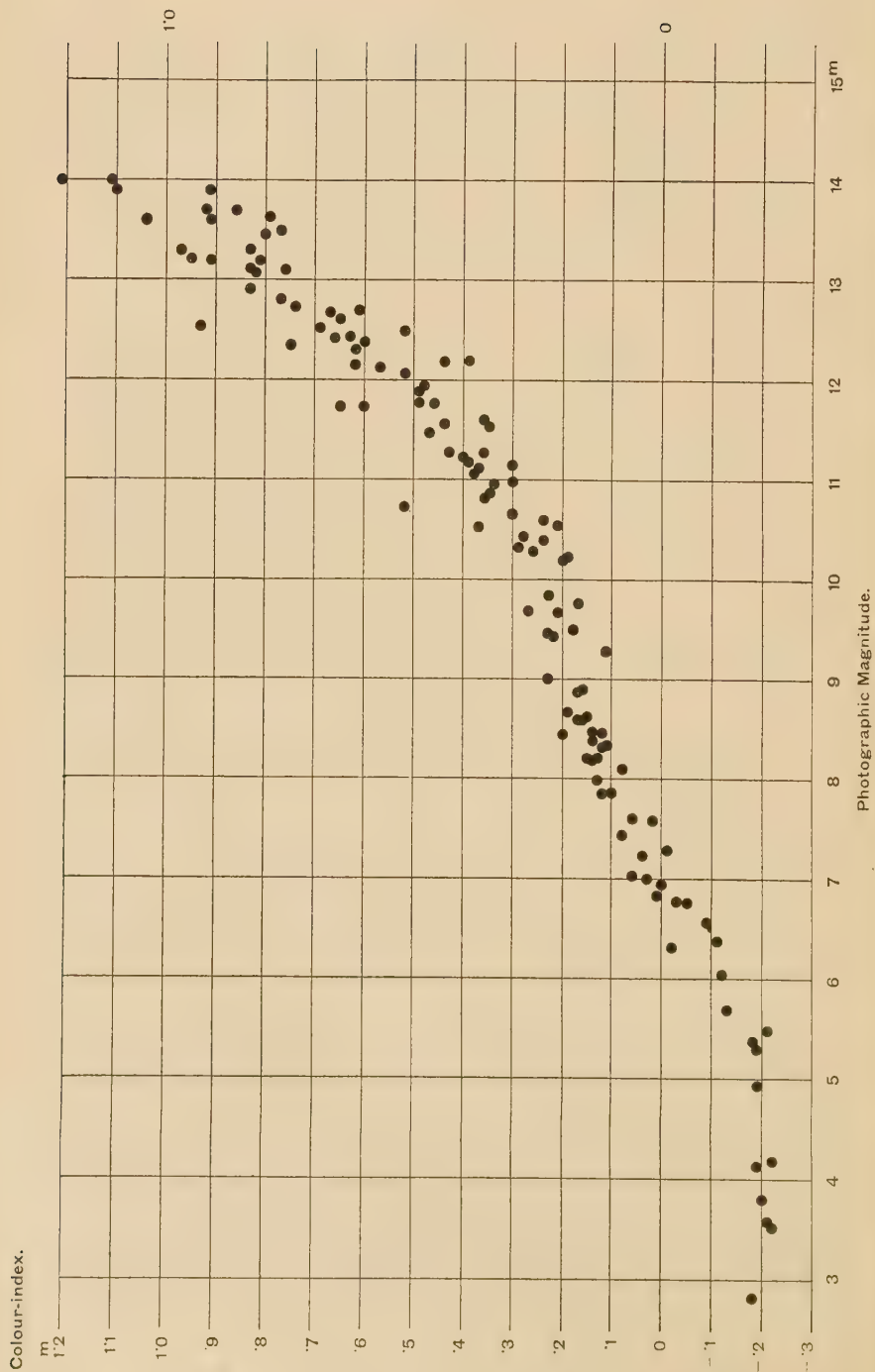
The next 9 diagrams, for colour-index between 0.75 and 1.20 , show the transition from the yellow "dwarfs" to the yellow "giants." In fact, there are only a few erratic "dwarfs" left. But there are several physical members of the Pleiades shown, and it is interesting to see how they turn more and more yellow with decreasing brightness.

The last 9 diagrams of the most yellow stars show very small proper motions with only three exceptions, of which two are faint physical Pleiades. The other 26 stars are evidently yellow "giants."

We thus find among these faint stars the same general picture of the relation between colour and proper motion as is well known for the naked-eye stars: Small proper motions for stars of extreme colours and great proper motions for stars of the same brightness and intermediate colour.

Concerning the physical members of the Pleiades only, we are specially interested in the connection between colour and brightness. We have here a fairly continuous series of stars of well-known colours, all standing practically at the same distance from us and covering an interval of at least 10 magnitudes.

Fig. 6 shows the relation in question. You see that all the physical Pleiades so far known belong to the main series. No yellow "giants" or "white dwarfs" are found. The next striking feature of the diagram is that stars of the same brightness have practically all the same colour. In this connection it should be remembered that the accuracy of the colours decreases with decreasing brightness of the



stars. In fact it is difficult to point out any physical member of the group showing a colour distinctly deviating from the normal one. Even when the colour of any individual object is found slightly too yellow, there remains the possibility of the star being double, in which case it would have to be shifted to the right on the diagram up to three-quarters of a magnitude. And of course there may be a few stars included which are not physical members of the group.

Looking at the central line of the diagram, it is seen that the 10 brightest and whitest stars between $2\frac{1}{2}^m$ and $5\frac{1}{2}^m$ do not show any sensible difference in colour. From this point the colour-index increases with decreasing brightness—first slowly, later more rapidly. Remembering that the colours here shown mainly depend on effective wave-lengths, it is possible, but not certain, that the secondary wave shown by the central line between 6^m and 10^m is caused by the disturbing influence of the head of the Balmer series, weakening the ultra-violet end of the spectrum.

At any rate the colour of the physical Pleiades changes neatly and gradually with the brightness. It is therefore interesting that this is less so in the case of the spectral class of the same stars. In fact, the Harvard classification of the physical Pleiades does not indicate any member as having a spectrum between A5 and F5. It simply means that the spectral classification does not correspond to a homogeneous colour-scale as is sometimes assumed.

The question now naturally arises, whether the brightness-colour line of the physical Pleiades is the same as for the ordinary main series. It is hardly possible to give a definite answer to this question at present.

If the central brightness-colour line of the physical Pleiades is identical with that of common stars, we may derive the parallax of the Pleiades simply by comparison of physical members of the group with common stars of known absolute brightness and colour. The best results of this kind may be expected by using the steeper part of the magnitude-colour line. From the mean values of absolute brightness and colour for 8 stars about like our sun I find in the way just mentioned the parallax of the Pleiades to be $0''.005$. A similar comparison with stars in the Hyades gives $0''.007$ for the parallax of the Pleiades. These two values do not agree very well and they are subject to revision. But this revision has to be rather considerable, if the apparent proper motion of the Pleiades is to be regarded as the reflected motion of our sun only. In that case the parallax of the Pleiades comes out to be $0''.0127$ against $0''.005$ as derived from comparison with the 8 stars just mentioned. The corresponding difference in absolute magnitude is about 2^m . If the difference is real, it means either that the Pleiades are whiter than common stars of the same absolute brightness or that they have peculiar motion of their own. The latter alternative appears rather improbable in view of the fact that both the direction of the proper motion and the radial velocity agree well enough with the motion of our sun. The deviation in the direction of the proper motion is about 9° . The observed mean radial

velocity is $+8$ km./sec. with a mean error of ± 2 km./sec.,* while $+9$ km./sec. corresponds to the motion of our sun.

It must be left to the future to solve this riddle. But the lack of yellow giants in the Pleiades gives this group a racial difference from the Hyades and Præsepe and it is not unlikely that this racial difference should manifest itself in other ways too. At any rate, the physical Pleiades behave according to a strict brightness-colour rule and they form a pure race.

One other way to make a fair estimate of the parallax of the Pleiades is presented by the calculation of dynamical parallaxes of double stars physically belonging to the group and showing measurable orbital motion. This method also gives a parallax of the order of magnitude of $0''.01$, but it does not solve the question.

No white dwarfs of Eddington's kind have as yet been found among the physical Pleiades. But if the parallax of the group is $0''.01$ or $\frac{1}{4}\frac{1}{10}$ of that of Sirius, the companion of Sirius would show the magnitude $8^m.5 + 5 \log 40 = 16^m.5$ if placed in the Pleiades. That is only half a magnitude from the limit of 16^m already reached and I hope to come still a little farther, where we may reasonably look for white dwarfs. But then we also need to know the colours of those faint stars, and these can only be determined by the aid of more powerful instruments than we have at Leiden.

After thus having considered the relation between colour and brightness for the physical members of the Pleiades, it is natural to ask how the rest of the stars, accidentally present in the same field, behave in this respect. That is shown in fig. 7, which is on the same scale as that of the physical Pleiades, the colour-brightness series of which is indicated by the full line. As you see, there are no faint stars as white as the brightest Pleiades.

One star is of particular interest, as its proper motion agrees with that of the group of the Hyades. Its photographic magnitude is $11^m.5$ and its colour-index $+1.13$. This colour-index corresponds to a star among the physical Pleiades of magnitude $14^m.3$. The difference in magnitude is $2^m.8$ and the corresponding parallax of the Pleiades is again $0''.007$ or $0''.008$, assuming that of the Hyades to be $0''.027$.

I cannot conclude a lecture on the Pleiades without saying a few words on the familiar nebulosity shown in this group. It has been announced by Slipher that the nebulosity shines by reflected light. In fact it has been found that the brightest patches of the nebulosity reflect about 1 per cent. of the light that a sheet of white paper in their place would reflect. We have another example of stellar light reflected by nebulosity in Nova Persei 2, and you will remember how a lower limit of its parallax could be derived by the angular velocity of the illumination compared with the real velocity of light. If there only were a spectroscopic binary among the brighter Pleiades, we could

* Nine stars in Voûte's second catalogue (*Ann. Bosscha Sterrenwacht*, 3, 1, 1928) give the mean value $+11.1 \pm 4.7$ km./sec. and eleven by J. Jung (*Astron. Mitt. Göttingen*, 17, 1914), not in Voûte's catalogue, give $+7.5 \pm 2.4$ km./sec. (m.c.). (In Table IX., l.c., the radial velocity of star 20c should read $+6.3$ km./sec.).

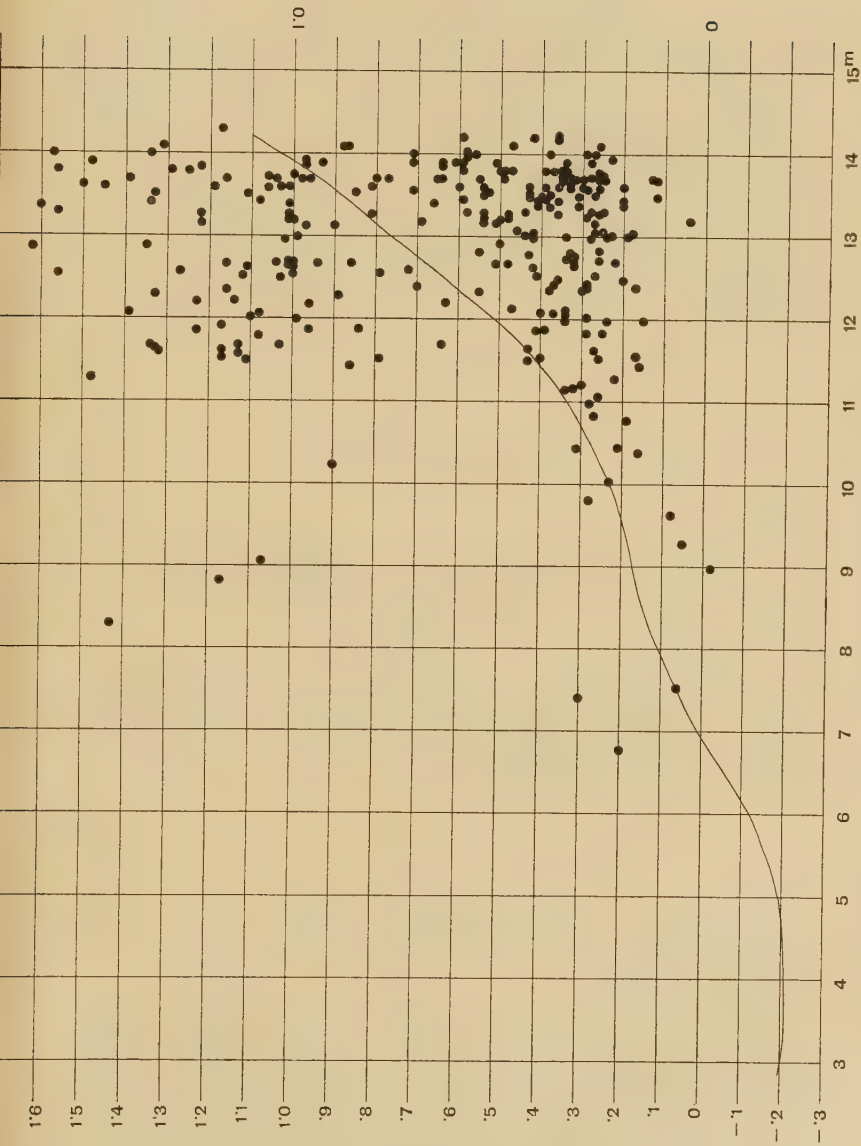


FIG. 7.

hope to find a lower limit for the parallax of the Pleiades in the same way, because an eclipsing binary, as which the star would be bound to appear in a certain plane, would send out more or less light at regular intervals in a period known from spectroscopic observations. The nebulosity would then appear striped and the distance between the stripes would in connection with the known period give a lower limit of the parallax. A period of one day would at a distance of 100 parsecs correspond to an angle between the stripes of $1''.73$ at the most. The order of magnitude would therefore be convenient. But I fear that this idea will remain Utopian.

Considering the question, whether the nebulosity shares the motion of the group or not, I think that this before long can be settled simply by comparing old and new photographs stereoscopically. Some of the patches of nebulosity appear as bright streaks nearly at right angles to the direction of the motion of the group. This is the ideal condition for stereoscopic vision.

As the present-day desiderata concerning the Pleiades I should like to mention :

1. A better determination of the radial velocity of the system by the aid of fainter physical members, showing spectra with sharper lines than do the brighter stars of the group.

2. Determination of colour-equivalents for physical members fainter than 14^m .

3. Exact comparison in colour with stars of known parallax.

4. Accurate determinations of angular distances between the outer physical members of the group, for later measurement of the apparent shrinking of the group in the course of time.

5. Examination also of the faint physical members for duplicity.

Finally you may ask me : Why do all this now, when proper motions of faint stars in the Pleiades with the accuracy presented here can be obtained in 100 years with one-tenth of the trouble now necessary ? My answer is that I cannot wait as long as that, and my only excuse : human impatience and curiosity.

The Number of Stars of Different Magnitudes in the Hyderabad Astrographic Catalogue. Fifth Paper. Zone -21° . By T. P. Bhaskaran, M.A.

1. A discussion of the number of stars of different photographic magnitudes in the Hyderabad Astrographic Zones Decl. -17° to -20° has been published in a series of four papers in the *Monthly Notices*.

On the completion of the original programme, the Nizamiah Observatory undertook the extension of the Catalogue to the three southern zones, -21° to -23° . The work has now been nearly finished, and vols. v. and vi. of the Astrographic Catalogue, containing the measures of rectangular co-ordinates in Zones -21° and -22° , have been issued.

As in the case of the previous zones, counts of star-images under different diameters have been made in Dec. -21° and the material is discussed in the present note.

2. The number of stars photographed in this zone is much greater than in any of the other zones of the Hyderabad section, the average per plate being 491. This was chiefly due to the more stringent conditions that were adopted during the later stages of the work for the acceptance of plates taken for the catalogue. The exposure times remained constant for all the photographs in this zone, so that the material collected can be considered fairly homogeneous.

3. The cumulative totals of the number of stars counted in each hour of R.A. are given in Table I., at the points for which $d = 50, 30, 20, 12$, and 8 (in units of $0''.15$). In the table N_x represents the density corresponding to forty square degrees for star-images whose diameter is not less than x . The galactic latitude for the middle of the hour is given in the last column.

TABLE I.

Cumulative Totals for Ten Plates in Each Hour of R.A. Zone -21° .

R.A. h	N_{50}^*	N_{30}^*	N_{20}^*	N_{12}^*	N_8 or All.	Gal. Lat.
0	55	342	737	1332	1522	$-83^{\circ}0$
1	58	416	863	1419	1536	$-77^{\circ}0$
2	80	482	917	1382	1654	$-64^{\circ}5$
3	84	566	1038	1622	1817	$-51^{\circ}0$
4	87	655	1291	2156	2510	$-37^{\circ}5$
5	116	884	1773	3157	3677	$-24^{\circ}5$
6	263	1642	3398	6532	8189	$-12^{\circ}0$
7	314	3019	7351	15190	18653	$0^{\circ}0$
8	126	1505	3596	7338	9667	$+12^{\circ}0$
9	133	1033	2179	3658	4278	$+23^{\circ}0$
10	101	917	1885	3145	3739	$+32^{\circ}0$
11	109	657	1345	2346	2691	$+38^{\circ}5$
12	101	474	980	1855	2214	$+41^{\circ}0$
13	72	522	1095	1858	2265	$+39^{\circ}5$
14	88	643	1276	2223	2662	$+34^{\circ}5$
15	92	481	917	1559	1922	$+26^{\circ}5$
16	73	373	806	1330	1546	$+16^{\circ}5$
17	156	794	1705	3013	3732	$+5^{\circ}0$
18	239	1991	4030	7194	8653	$-7^{\circ}0$
19	133	1078	2329	4386	6407	$-19^{\circ}5$
20	99	781	1715	3281	4032	$-32^{\circ}5$
21	86	583	1276	2272	2693	$-46^{\circ}0$
22	57	513	1109	1876	2155	$-59^{\circ}5$
23	57	408	797	1264	1490	$-73^{\circ}0$

4. Now, taking the total star density for all the hours of R.A., under the different headings, the magnitudes have been derived on the Chapman and Melotte scale from the data given in the table in *M.N.*, 75, 145. These are shown in the following table:—

TABLE II.

Diameter.	50.	30.	20.	12.	8.
C. and M. Mag.	9.21	11.27	12.17	12.90	13.16
Calculated	9.50	10.98	11.92	12.86	13.45
Difference	-.29	+.29	+.25	+.04	-.29

In the row "Calculated" are given the magnitudes obtained from the formula $16.08 - 0.93\sqrt{d}$, which has been derived independently for this zone. The values of the constants are in good agreement with those obtained for the previous Hyderabad zones. A better formula seems to be $m = 19.74 - 1.22\sqrt{d} + 20$, which brings the magnitudes of the fainter stars more in accordance with the observed values.

5. In order to deduce a value for the galactic condensation in this zone, the star density in the areas with galactic latitudes less than 20° is compared with the density in areas with latitudes greater than 40° . The resulting mean values are given in Table III. for each of the points at which counts have been made.

TABLE III.

Diameter.	Ptg. Mag.	Gal. Condensation.
50	9.21	2.6
30	11.27	3.1
20	12.17	3.4
12	12.90	3.9
8	13.16	4.3

The coefficient in the case of the fainter stars is slightly greater than the average value, due to the number of open clusters which occur in the region R.A. 6^h and 7^h . Grouping the hours according to their galactic latitudes, we obtain the coefficient of condensation per degree per unit of $\log N$ equivalent to .0031.

6. We now proceed to remove the effect of the galactic concentration from the data of Table I., following the standard method adopted by Professor Turner in several of his papers (see *M.N.*, 78, 54). $\log N_0$, the logarithm of star density, cleared of galactic effect, has been obtained from the actual star density $\log N_\gamma$ by the formula

$$\log N_0 = (1 + .0025\gamma) \log N + K_\gamma.$$

The mean value of $\log N_0$, corresponding to each of the selected points, is shown in Table IV. The mean results for zones -17° to -20° are also given for comparison.

TABLE IV.
Values of log N₀.

Zone.	diam. 50.	30.	20.	12.	8.
-21	0.70	1.63	1.98	2.24	2.33
-17	0.66	1.54	1.87	2.15	2.33
to					
-20					

7. The residuals, after subtracting the mean values of log N₀, are given in Table V. (unit 0.01 in log N₀).

TABLE V.
*Observed Residuals after correcting for Galactic Effect.
Hyderabad Astrogaphic Zone -21°.*

R.A.	50.	30.	20.	12.	8.	R.A.	50.	30.	20.	12.	8.
oh						h					
-9	-6	-1	+4	+2		12	+11	-8	-8	-4	-4
1	-7	+2	+1	+4	0	13	-6	-4	-4	-5	-4
2	+9	+6	+4	-1	-1	14	-1	+1	-2	-2	-2
3	+7	+8	+3	-1	-5	15	-8	-24	-29	-31	-30
4	+2	+4	+2	0	-2	16	-32	-51	-51	-55	-57
5	0	+1	-2	-2	-4	17	-9	-31	-32	-34	-33
6	+20	+9	+7	+10	+11	18	+11	+12	+8	+8	+7
7	+19	+25	+28	+34	+34	19	0	+2	+2	+5	+13
8	-13	+5	+10	+15	+19	20	+2	+6	+8	+13	+14
9	+5	+7	+6	+4	+2	21	+7	+6	+9	+11	+10
10	+3	+14	+13	+11	+10	22	-8	+9	+12	+12	+10
11	+12	+5	+5	+5	+3	23	-8	0	-1	-3	-4

8. These residuals are next analysed harmonically, and the results are tabulated below for the different values of log N₀.

TABLE VI.
Harmonic Analysis of the Residuals.

log N ₀ .	Sin θ.	Cos θ.	Sin 2θ.	Cos 2θ.	Sin 3θ.	Cos 3θ.	Sin 4θ.	Cos 4θ.
0.70	+0.062	-0.014	-0.037	-0.004	0.000	-0.071	+0.020	+0.032
1.63	+0.110	+0.029	-0.096	+0.028	-0.010	-0.057	+0.051	-0.022
1.98	+0.110	+0.034	-0.119	+0.039	-0.017	-0.047	+0.059	-0.025
2.24	+0.113	+0.035	-0.141	+0.039	-0.027	-0.045	+0.079	-0.020
2.33	+0.102	+0.031	-0.153	+0.029	-0.030	-0.046	+0.098	-0.023
1.02	+0.022	+0.025	-0.071	+0.024	-0.020	+0.018	+0.043	-0.028
1.00	+0.022	+0.025	-0.070	+0.024	-0.020	+0.018	+0.042	-0.027

The row below the horizontal line is obtained by subtracting the mean of the first two rows from the mean of the other three; the last row gives the rate of change of the harmonics for each unit of log N₀.

9. We now consider the phases of the first harmonic, which may be expressed in the form $\cdot 033 \cos (\theta - 41^\circ)$. The value obtained from the spiral of maximum obscuration, $\alpha = 247^\circ - 3.66\delta$, is 324° . The discordance $O - C$ is in this case $+77^\circ$, which is larger than that found for the neighbouring zones. We may compare the phases of the first harmonic with the corresponding values for other southern areas, counts for which are now available.

-15° to -19°	$\cdot 079 \cos (\theta - 12^\circ)$
-20°	$\cdot 038 \cos (\theta - 101^\circ)$
-21°	$\cdot 033 \cos (\theta - 41^\circ)$
-25° to -31°	$\cdot 021 \cos (\theta - 131^\circ)$

There seems to be a rapid change in the phase between -15° and -25° , as originally remarked by Professor Turner, but the progression is in no way uniform.

Among the other harmonics, it is noticed that the coefficients of $\sin 4\theta$ and $\cos 4\theta$ show a remarkable agreement with the mean values for these regions (*vide M.N.*, **85**, 615, 1925).

Similar counts are in progress for the remaining two zones, -22° and -23° , of the Hyderabad section of the Astrographic Catalogue.

*Nizamiah Observatory,
Hyderabad, Deccan, India :
1929 March.*

On Multiple Solutions in the Determination of Orbits.

By H. C. Plummer.

1. In the problem of determining the orbit of a planet, reduced to its simplest terms, let S, E, P denote the corresponding positions of the Sun, the Earth and the planet. In this triangle let $SE = R$, $SP = \sigma$, $EP = \rho$, $SPE = z$, $SEP = 180^\circ - \delta$. Thus

$$\frac{\sigma}{\sin \delta} = \frac{\rho}{\sin (\delta - z)} = \frac{R}{\sin z}, \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

and the inequalities

$$0 < z < \delta < \pi \quad \cdot \quad \cdot \quad \cdot \quad (2)$$

must be observed. The side R and the angle δ may be taken to be known ; in addition there is an equation of the form

$$b\rho = R^4 \left(\frac{1}{R^3} - \frac{1}{\sigma^3} \right), \quad \cdot \quad \cdot \quad \cdot \quad (3)$$

where the value of b must be determined from observations. Its meaning can be expressed by

$$-b = \frac{v^2 \cot \omega}{k^2 \sin T} R^3,$$

where v is the apparent angular velocity of the planet, k (the Gaussian constant) is essentially the Earth's mean motion, ω is the radius of the small circle which osculates the apparent path of the planet, and T is the perpendicular arc drawn from the Sun to the tangent (great circle) to the apparent path. This arc is to be reckoned negative, and therefore b is positive and $\sigma > R$, when the Sun and the osculating circle lie on opposite sides of the tangent great circle.

2. From (3), written in the form

$$R^4 = \sigma^3(R - b\rho),$$

it follows by (1) that

$$\sin^4 z = \sin^3 \delta [\sin z - b \sin (\delta - z)], \quad . \quad . \quad (4)$$

which can be at once reduced to the normal Gaussian form, but this is unnecessary, since the object is merely to examine the nature of the appropriate solution.

A ready means of doing this is to consider the line

$$\sin^3 \delta [y(1 + b \cos \delta) - bx \sin \delta] = 1 \quad . \quad . \quad (5)$$

in connection with the curve whose polar equation is

$$r \sin^4 \theta = 1, \quad . \quad . \quad . \quad (6)$$

for the intersections of the straight line with this standard curve give corresponding values of θ , which are clearly the values of z which satisfy (4).

3. It is evident that the curve (6) consists (fig. 1) of two branches symmetrically separated by the axis of x , and that the branch below the axis of x ($\pi < \theta < 2\pi$) is not relevant to the problem and can be ignored. In the next place, the form of the upper branch shows by inspection that a straight line will cut it in one or in three points. Now $z = \delta$ is an obvious solution of (4). Apart from this ($\rho = 0$) there must be the solution which is sought. Hence the intersections of the line and curve must be three in number, of which two coincide when the line is a tangent. Let $A(\theta = \delta)$, $P_1(\theta = z_1)$, and $P_2(\theta = z_2)$ be the three distinct intersections. Since, by (2), $\delta > z$, the order of these points from left to right may be either AP_1P_2 or P_1AP_2 , but not P_1P_2A . If it is P_1AP_2 , $\theta = z_2$ is the only possible solution; if it is AP_1P_2 , both $\theta = z_1$ and $\theta = z_2$ are possible solutions, and further information is required to discriminate between them.

4. The relevant half of the curve (6) has a point of inflexion on either side of the axis of y , and according to its slope the line (5) must pass somewhere near one of these points, with at least two of its three intersections on the same side. Which side of the y -axis these shall fall, and whether the line slopes upwards or downwards towards the right, is decided by the sign of b . If b be positive, the normal case of a superior planet ($\sigma > R$), not only $\delta > z$ but $\sin \delta > \sin z$. Now the ordinate of the curve is $y = (\sin \theta)^{-3}$, and therefore the ordinate at $A(\theta = \delta)$ is less than the ordinate at $P(\theta = z)$. In this case the line slopes upward to the right, and at least two intersections lie to the right of the axis

of y . If b be negative, $\sigma < R$ and $\sin \delta < \sin z$, though $\delta > z$. Hence the ordinate at A must be greater than the ordinate at P, and the line slopes downward to the right. In this case it is clear that $\frac{1}{2}\pi < \delta < \pi$.

5. At a point of the curve (6) let ϕ be the inclination of the tangent to the radius vector, so that

$$\cot \phi = \frac{1}{r} \cdot \frac{dr}{d\theta} = -4 \cot \theta.$$

The inclination of the tangent to the axis of x is therefore given by

$$\frac{dy}{dx} = \tan(\theta + \phi) = \frac{3 \tan \theta}{4 + \tan^2 \theta}.$$

Parallel tangents are thus given by

$$\tan \theta_1 / (4 + \tan^2 \theta_1) = \tan \theta_2 / (4 + \tan^2 \theta_2),$$

or $\tan \theta_1 \tan \theta_2 = 4$, and these coincide at the points of inflexion, given

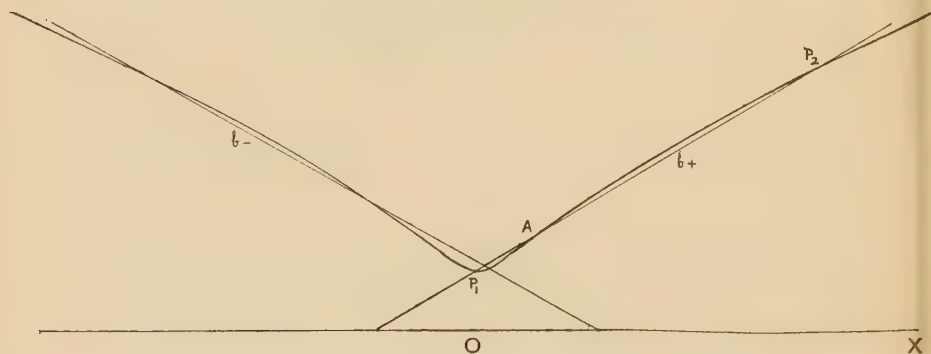


FIG. 1.

by $\tan \theta = \pm 2$. The equation of the tangent at the point $(\sin^{-4} \theta \cos \theta, \sin^{-3} \theta)$ is

$$y(4 + \tan^2 \theta) - 3x \tan \theta = 1/\sin^3 \theta \cos^2 \theta \quad (7)$$

6. The criterion for a single valid solution of the problem is now easily obtained. The intersection $A(\theta = \delta)$ must be the middle arc of the three points in which the straight line (5) crosses the curve (6). It follows by inspection of the curve that the tangent at A must lie between the line and the radius vector to the point. Now, if χ be the inclination of the line to the x -axis,

$$\tan \chi = \frac{b \sin \delta}{1 + b \cos \delta},$$

and its inclination to the radius vector ($\theta = \delta$) is therefore given by

$$\tan(\delta - \chi) = \frac{\sin \delta}{b + \cos \delta},$$

while the inclination of the tangent to the radius vector is given by

$$\tan \phi_1 = -\frac{1}{4} \tan \delta.$$

The criterion sought is therefore

$$|\tan \delta| < \left| \frac{4 \sin \delta}{b + \cos \delta} \right|$$

or

$$|b + \cos \delta| < 4 |\cos \delta|,$$

since $\sin \delta$ is positive. But b and $\cos \delta$ have the same sign, whether the point A lies to the right or left of the origin. Hence $|b| < 3 |\cos \delta|$, or the valid solution is unique if

$$b(b - 3 \cos \delta) < 0, \quad . \quad . \quad . \quad . \quad (8)$$

whether b is positive or negative.

7. The line (5) passing through $A(\theta = \delta)$ coincides with the tangent (7) at A when $b = 3 \cos \delta$. In this case

$$\begin{aligned} b\rho &= R^4 \left(\frac{1}{R^3} - \frac{1}{\sigma^3} \right) = 3\rho \cos \delta \\ &= 3[\sigma \cos(\delta - z) - R], \end{aligned}$$

or

$$4 = \frac{3\sigma}{R} \cos(\delta - z) + \frac{R^3}{\sigma^3} \quad . \quad . \quad . \quad (9)$$

This is the polar equation giving the corresponding position of the planet $(\sigma, \delta - z)$, the pole being the Sun and the radius vector of the Earth the initial line. It is the *limiting curve* of Charlier, which plays a part in the discrimination between a single and double solution. The locus $b = 0$, or the circle $\sigma = R$, plays a similar part.

8. On the other hand, the line (5) may be a tangent at $P(\theta = z)$, the points P_1 and P_2 coinciding. Then (5) coincides with (7), written in the form

$$y(5 + 3 \cos 2z) - 3x \sin 2z = 2/\sin^3 z,$$

and this requires

$$\begin{aligned} 2 \sin^3 \delta (1 + b \cos \delta) &= (5 + 3 \cos 2z) \sin^3 z, \\ 2b \sin^4 \delta &= 3 \sin 2z \sin^3 z. \end{aligned}$$

Hence, when b is eliminated,

$$2 \sin^4 \delta = [5 \sin \delta + 3 \sin(\delta - 2z)] \sin^3 z,$$

or

$$\sin^4 \delta = [\sin \delta + 3 \sin(\delta - z) \cos z] \sin^3 z.$$

By (1) this can be written

$$\begin{aligned} \sigma^4 &= (\sigma + 3\rho \cos z)R^3 \\ &= [4\sigma - 3R \cos(\delta - z)]R^3, \end{aligned}$$

or finally

$$4 = \frac{3R}{\sigma} \cos(\delta - z) + \frac{\sigma^3}{R^3} \quad . \quad . \quad . \quad (10)$$

This defines the locus of the planet in the same polar co-ordinates $(\sigma, \delta - z)$ as those occurring in (9), of which it is clearly the inverse with respect to the circle $\sigma = R$. It is the *singular curve* of Charlier.

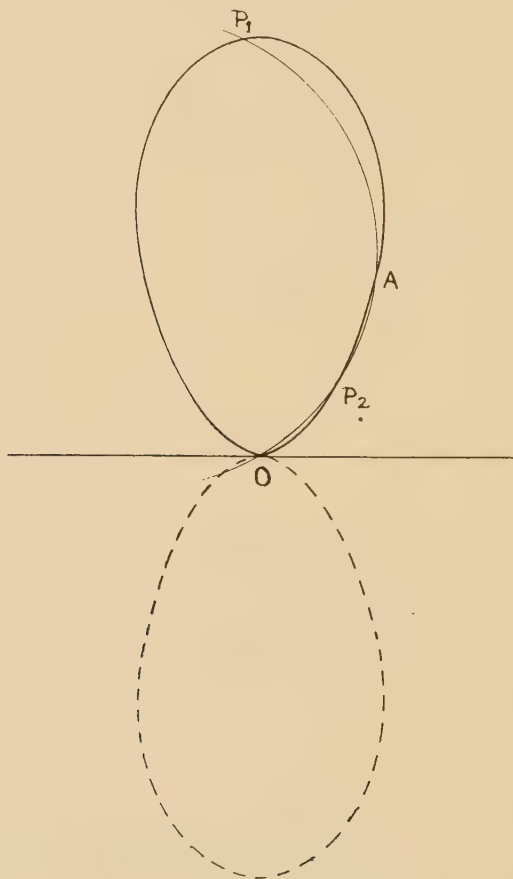


FIG. 2.

Two apparently valid solutions of the problem coincide when the planet lies on either of the curves* (9) or (10), but there is an essential difference between the two curves. The intersections of (5) and (6) being A, P_1 , P_2 in order, the coincidence of P_1 with A leads to the limiting curve (9) which divides the region giving rise to one solution

* For a figure of these curves and an alternative discussion, see the author's *Dynamical Astronomy*, ch. vii.

only from the region giving rise to two. On the other hand, the coincidence of P_1 with P_2 leads to the singular curve (10), and only one solution is then possible. But the region on either side of this curve gives rise equally to an ambiguous choice of solutions.

9. The nature of the possible solutions of the problem can also be exhibited by substituting $1/r$ for r , and replacing the line (5) by its inverse, the circle

$$\sin^3 \delta [y(1 + b \cos \delta) - bx \sin \delta] = x^2 + y^2, \quad . \quad . \quad (11)$$

and (6) by its inverse, the curve

$$r = \sin^4 \theta. \quad . \quad . \quad . \quad . \quad (12)$$

The latter (fig. 2) consists of two simple loops, of which the upper alone ($0 < \theta < \pi$) is relevant to the question. While the advantage presented by the straight line is lost, the infinite branches of (6) are avoided, and it is equally evident that a circle will intersect a simple loop of (12) in 0, 2, or 4 points. The actual number must be 4, since the circle (11) passes through the origin (corresponding to the points at infinity on (5) and (6)), through the point $\theta = \delta$, and through the point $\theta = \pi - \delta$, the solution sought. Thus there must be a fourth intersection, which again provides a valid solution if the inequality (2) is satisfied, and not otherwise.

Discussion of Observations of Three Long-period Variables, made by A. N. Brown, M.A. No. 1, RT Cygni, 1908-1928. By Mary A. Blagg.

[These observations down to 1927 June have been printed in detail in *M.N.*, in 7 papers: 1908 May, 1910 Jan., 1911 Mar., 1912 Mar., 1917 Mar., 1922 May, 1927 June.]

The discussion, undertaken at the observer's request, of these carefully made and recorded observations, has proved very interesting. Forty cycles of the star's main variation were completed in the series, occupying nearly 21 years.

It is well to state at once that no actual maximum or minimum date can be fixed precisely by observation in any single period. In the dates suggested in the above papers there is always a doubt of at least 4 days, and usually of much more. The observer, in mentioning this fact (1917), says that the star appears to remain at maximum and minimum light for several days. This must inevitably *appear* to be the case. In a sine curve of the dimensions of this star's variation the difference in magnitude for 6 days on either side of maximum and minimum is less than 0.04. Even if the magnitude were deduced to 0.01 m. instead of (as here) to 0.10 m., such differences as these would be swamped by "accidental errors." The plan of taking as date of

maximum or minimum the apparent middle point of the apparently flat portion could be precisely accurate only if the curve rose and fell with perfect symmetry and the observations formed a continuously unbroken series—an impossible condition. It follows that the actual length of any individual cycle cannot be accurately determined. But when (as in this case) we have a long series of cycles, the number of which is absolutely certain, the *average* length of the period can be found with little difficulty. Inspections of these observations suggested a period of 191 days, and they were therefore divided into cycles of that length, each cycle being subdivided into 12 “portions” of nearly 16 days each. The means in magnitude for each portion were tabulated, taking the first and second sets of 20 cycles separately. The result gave two almost identical mean curves, the phases of the second being at an average distance of about 4 days ahead of the first, implying a needed correction to the assumed period of +0.2 days. The observations were therefore rearranged in cycles of 191.2 days, and this time there was no sensible difference of phase between the two resulting curves. The mean curve obtained from the whole of the 40 cycles worked out thus (the central date of portion 1, cycle 1, being J.D. 2417958) :

Portions.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Mean mag. }	7.98	8.86	9.85	10.67	11.05	11.05	10.58	9.67	8.64	7.69	7.40	7.53

Analysis of these figures gave the following formula :—

$$1.9 \cos (\theta - 131^{\circ}.6) + .05 \cos (2\theta - 2^{\circ}) + .05 \cos (3\theta - 239^{\circ}),$$

forming the curve :

$$8.01, 8.86, 9.86, 10.66, 11.05, 11.03, 10.59, 9.70, 8.60, 7.74, 7.39, 7.51.$$

This is in excellent agreement with the observed curve, and the smallness of the coefficients of the second and third terms shows how nearly the curve approximates to the simple sine form. These small terms indicate a shorter interval from minimum to maximum than from maximum to minimum—*i.e.* the rise is slightly more rapid than the fall. The mean magnitude of the star is about 9.25, the average length of the rise about 91 days and of the fall about 100 days. The *mean* range of variation is not quite 4 magnitudes, but the range varies greatly. The smoothness and regularity of the mean curve are the more remarkable because individual cycles seem to differ considerably (*a*) in general brightness, (*b*) in rapidity of rise or fall, (*c*) in duration of apparent maximum or minimum. Fluctuations in the *length* of the period, which have usually formed the chief subject of investigation, could here be found only indirectly through the subsidiary variations of magnitude throughout the star's course, with which such fluctuations are intimately connected—whether as cause or effect is here immaterial. These variations may in themselves be often of greater interest than the fluctuations in length of period connected with them. In order to examine any such subsidiary variations (other than any which are submultiples of the main cycle), the deduced magnitude of each individual observation was

compared with the magnitude proper to its position in the curve. The differences (O-C) were then arranged in the same manner as was employed for obtaining the mean curve itself. (The signs “+” and “-” are used in their true arithmetical sense, so that *negative* residuals in magnitude imply greater brightness. In the papers in *M.N.* mentioned above the meaning of the signs was reversed, “+” meaning “brighter.”)

These residuals brought out interesting and remarkable results. In addition to a quite definite but far from simple systematic fluctuation in general magnitude continued throughout, there are often *small* fluctuations, backwards and forwards, round about the time of maximum (noticeably in 1915 Oct.-Nov.), which are, I think, of doubtful reality. But there are several cases of great and rather sudden increase or decrease of brightness, reaching about a magnitude above or below the normal, and persisting for a month or so at a time, then quickly subsiding, which strongly suggest some real interference with the star's ordinary course of variation. The most conspicuous instances are given in the following table:—

TABLE I.

(a) *Great Relative Brightness.*

Date.	Cycle.	Portions of Curve.	Mean Sid. Time of Obs.	Mean Resid.	No. Obs.	Brightest Recorded Mag.
(1)*1910 Jan. 19 to Feb. 15	4(5)§	11 to 1	^h IV	^m -0.9	11	6.6 (Jan. and Feb.)
(2) 1913 Jan. 26 to Mar. 2	10	8 to 10	V	-1.2	6	6.5 (Feb.)
(3) 1914 Feb. 15 to 28	12	8 and 9	(2) V; (1) XVI	-1.1	3	7.0 (Mar.)
(4) 1919 June 21 to July 15	22	11 and 12	XVII	-0.9	4	6.3 (June)
(5)*1920 Feb. 8 to Mar. 18	24	1 to 4	(2) IV; (2) XVII	-1.0	4	7.4 (Jan.)
(6)†1922 July 18 to 30	28	9 and 10	XVIII	-1.0	4	6.6 (July)
(7) 1926 Mar. 9 to May 9	35(36)§	9 to 1	XI	-1.1	12	6.3 (Mar. and Apr.)

(b) *Great Relative Faintness.*

(8) 1910 Dec. 12 to 1911 Jan. 31	6	8 to 11	II	+1.3	14	8.1 (Feb.)
(9)‡1917 Oct. 23 to Dec. 6	19	9 to 12	O	+1.1	10	8.2 (Dec.)
(10) 1918 Oct. 24 to Nov. 15	21	7 to 9	O	+1.0	6	7.3 (Dec.)
(11) 1926 Oct. 23 to Dec. 15	36(37)§	11 to 2	O	+0.9	9	7.9 (Oct. 8 and 18)

The place of maximum, which occurs, normally, late on in portion 11, is markedly *earlier* in cycles 10, 12, 28, and 35. In 4 it is greatly lengthened in duration, and in 23 the brightening came too late to bring

* Followed by long gap.

† Preceded by long gap.

‡ Followed by continued relative faintness up to end of January. This example involved an actual *backward movement*, suddenly, in mid-October.

§ The figures in brackets mean that the observations in question continue into cycles 5, 36, 37 respectively, but the maximum is in the non-bracketed cycle.

either an accelerated or a brightened maximum, but it brought a second pseudo-maximum in February. Maximum is markedly *late* in cycles 6, 19, 21, and 23; but in 36 it was apparently cut off prematurely by the sudden faintness starting in portion 11.

The figures in the table do not seem to indicate any very regular intervals between these several occurrences, but they do display certain rather striking phenomena. (1) In both (a) and (b) they occur almost entirely within the same half of the cycle—in portions 8 to 1—while in (a) they are always in the 6 months from mid-January to July, and in (b) always in the $3\frac{1}{2}$ months from mid-October to the end of January. The mean sidereal time of the observations differs in a definite manner between (a) and (b). Upon this, it seemed well to look for “seasonal” effects, in the hope of eliminating them before investigating the systematic fluctuations alluded to above. The residuals were tabulated first by months and then by sidereal hours. Table II. gives the result by months. The whole series was divided into two sections: 1908–18 and 1919–28. The first contains almost exactly 21 cycles and the other almost exactly 19 cycles, thus spreading the months evenly over the whole of the cycle. The unit is one hundredth of a magnitude.

TABLE II.

Sections.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
I.	+8	-23	(+2½)	+4	-4	-10	-16	-10	-1½	+9	+22	+1
II	+8	-14	-11	+6	-2	-3	-14	-7	-6	-1	+19	+1
Weighted mean }	+8	-19	(-5)	+5	-3	-7	-15	-8	-4	+4	+20	+1
Average S.T. (app.) }	2.5	4.6 (var.)	(Very var.)	(Very var.)	14.4	16.6	18.3	19.0	21.2	23.4	(Very var.)	0.6

TABLE III.

Hours.	O.	I.	II.	III.	IV.	V. to VII.	VIII. to X.	XI. to XIII.	XIV.	XV.	XVI.	XVII.	XVIII.	XIX.	XX. to XXI.	XXII. to XXIII.	XXIII.
Sec. I.	+11	+18	+9	-3	(0)	(-47)	..	+6	+10	-1	-8	-9	-15	-11	-6	+6	0
„ II.	+12	+14	+4	+16	-10	(-5)	(-1)	-1	0	-7	-12	-11	-7	-13	(+2)	-9	+1
Weighted mean }	+11	+16	+7	+6	-5	(-27)	(-1)	+2	+6	-4	-10	-10	-12	-12	-3	-3	+1
Months	{ Sept. to Jan.	Aug. to Jan.	Aug. to Jan.	Sept. to Feb.	Nov. to Feb.	Jan. to Mar.	Feb. to Apr.	Feb. to May.	Jan. to May.	Jan. to June.	Feb. to June.	Apr. to July.	May to Aug.	June to Sept.	July to Nov.	Aug. to Nov.	Sept. to Dec.

Brackets indicate paucity of data.

In March, the hours of observation are almost entirely different in the two sections.

The figures in most cases are *small* enough to be possibly accidental, especially having regard to the *larger* systematic differences in general magnitude which were conspicuously shown between one year (and one cycle) and another. But the agreement between the two sections

in both tables can scarcely be accidental, and that between the two tables is fairly close. The accordance and symmetry in Table II. are rather better than in Table III., but neither is really independent of the other; the S.T. varies with the months, though not quite evenly. The large means for February, July, November, and December are in part accounted for by the special events shown in Table I.; but, regardless of that consideration, the February mean is brighter than that of January in seventeen out of the twenty years in which both months have observations, while November is fainter than September in sixteen out of twenty years. An analysis was made of each individual month with each S.T. hour, and this showed that the negative sign predominates for *every* observed hour in June, July, and August, and very nearly so in February, May, and September, and the positive sign nearly so in November to January; while for the hours XVII. to I. the negative sign predominates in each month except from October to January. In this analysis the mean monthly figures are greater and the average "errors" less than those for the individual hours. This may, however, be due in part to the greater number of the hours, involving a smaller average number of observations for each.

The causes underlying these results are of a very varied and complicated character. Atmospheric effects are among the most uncertain. Those due to the H.A. of the star (or S.T.) vary, in the first place, with its altitude and orientation, but the effects of the latter are complicated by the use of different comparison stars at different times (some of which are "above" or "to the right of" it at one time, and *vice versa* at another), and perhaps also by variations in its colour. The latter two points depend on the star's place in its cycle of variation. Tables were drawn up analysing the relations of magnitude and also of the position in the cycle with each S.T. hour. In the first case, the "+" sign predominated for magnitude 9.7 to 10.6 at all hours. In the second, the "-" sign predominated *near minimum* for the hours XXIII. to I., thus reversing the sign shown for those hours for the whole cycle in Table III. But in these individually analysed tables the number of observations in most of the hour-sections is too small to have much weight.

The star's R.A. (1900) being XIX.^h 40^m 48^s, its altitude would differ most between nearly XX.^h and nearly VIII.^h Table III. shows no apparent difference here. In between these times the differences in configuration are opposite in character on each side, while the altitude varies identically on both sides. The sum of the mean residuals from XI. to XVII. is - 16, and they brighten as the star rises higher; the sum of those from XXIII. to IV. is + 36, and they brighten as the star sinks lower. This suggests that configuration has much, and altitude little or nothing, to do with these variations. But there is at least one other probable cause of variation to be considered, viz. the effect of brighter or darker skies, due partly to the Sun's distance below the horizon and partly to the phases of the moon. Table IV. gives an analysis of the residuals in connection with the former point, but the classification was made rather roughly, and no account was

taken of moonlight. The figures cannot be regarded as of great value, but they do seem to indicate that the star appeared relatively fainter in a dark sky, though not with complete consistency. Here again the results are not wholly independent of those in Tables II. and III., as the sidereal time changes each night with the sinking and rising of the Sun, and in the summer months a clear sky is never very dark.

TABLE IV.

(a) *After Sunset.*

Sun below Horizon.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Weighted Mean.	Ditto, omitting May to Aug.
<18°	+ 1	-27	(+12½)	..	- 3	-8	-15	- 3	-11	..	(+37½)	+18	- 3	+
18-25°	+17	-35	(-25)	+21	- 3	..	..	-33	+ 8	-5	+ 8	+14	- 2	+
>25°	+28	(-60)	-17	0	..	..	..	..	- 4	+5	+27	+14	+ 3	[+]

(b) *Before Sunrise.*

<18°	(-30)	(-47½)	..	(+ 8)	(-15)	-7	(-33)	(-5)	+10	..	..	..	-12	-10
18-25°	..	+10	(+ 3)	- 1	(-10)	..	..	(+30)	(+10)	..	..	(-10)	+ 5	+
>25°	..	(+20)	(+40)	+17	..	..	..	..	(-20)	..	..	..	+15	[+10]

Brackets indicate that the observations occurred in less than 5 separate years.

These three tables (and the more detailed ones mentioned), while supplying clear and on the whole consistent evidence for a regular seasonal variation occupying (roughly) six months or twelve sidereal hours—but of unequal amount in the two half-years—do not make possible a formula of correction applicable to all conditions. It is impossible so to dissect the individual observations as to discover the effect of configuration for each comparison star. There are, however, certain lines on which corrections can be applied, having careful regard to the partial exceptions shown in the detailed analyses. All three tables give similar figures for May or June to August or early September = XV.^h to XX.^h (approx.) when the Sun is not more than about 25° below the horizon; and all agree pretty well for November to January = 0^h to II.^h (approx.) when the Sun's distance below the horizon does not appear to make any certain difference. Again in February, III.^h to VII.^h and XV.^h to XVI.^h, the evidence is consistent. In the months of March and April, late September and October, the effects are much more uncertain and less consistently shown in the detailed tables mentioned above. The seasonal corrections given in Table V. and in the diagram were not applied until after the investigation of the other forms of fluctuation, so that they cannot have affected the figures in the "C" columns.

Owing to these seasonal effects, *short* genuine periodic fluctuations could not safely be sought. Longer ones were sought as follows. At

points near the middle and end of each period, the residual (relative) magnitude was found and plotted. Inspection of the curve thus formed suggested fluctuations of *about* 6, $7\frac{1}{2}$, 12, 15, and 2 or 3 periods in length, and (less clearly) of $4\frac{1}{2}$ and 9 or 10 periods. Analysis of the mid-period figures gave amplitudes of from 10 to at least 20 (0.1 and 0.2 magnitude) for 6-, $7\frac{1}{2}$ -, $2\frac{1}{2}$ -, and $3\frac{1}{2}$ -period fluctuations. Corrections to the length of these waves were obtained by dividing the series into two parts, each of 20 periods, which made the "6"- and " $2\frac{1}{2}$ "-period waves each a little longer, and the " $3\frac{1}{2}$ " period one a little shorter. The correction obtained for the $7\frac{1}{2}$ -period wave would make it a little longer, but the difference was rather uncertain and so small as not to seem worth applying. The 15-period wave is too long to be effectually corrected in only 40 periods, but it seemed clearly too short, and one of 16-period length was adopted instead. Good evidence was also found for the $4\frac{1}{2}$ -period wave. In Table V. these six fluctuations are given, and the sum of them is compared with the observed residuals, at points corresponding with the middle of portion six in each 191.2-days period. (See mean curve above.)

TABLE V.

Period.	Observed Resids. (Portion 6).	Same corrected for "Season."	C ₁ .	C ₂ .	C ₃ .	C ₄ .	C ₅ .	C ₆ .	Sums of C's.	O-C.	Positions.	
											Min.	Max.
1	(+14)	(+20)	-11	+13	+12	+4	0	+1	+19	+1	Late?	..
2	+22	+22	+7	+13	-10	-5	+4	+10	+19	+3	Early?	..
3	(0)	(+7)	+7	+1	+4	-11	+7	+2	+10	-3	Late	..
4	-19	-19	-11	-13	+3	-9	+9	-9	-30	+11	..	Late?
5	-40	-36	-1	-13	-10	-2	+10	-5	-21	-15	..	Early.
6	+54	+46	+12	-1	+12	+7	+9	+7	+46	0	V. late.	Late.
7	(+10)	(+14)	-5	+12	-10	+11	+7	+8	+23	-9	..	n.o.
8	(+4)	(+7)	-9	+14	+3	+8	+3	-5	+14	-7	..	n.o.
9	+14	+18	+10	+2	+4	0	0	-10	+6	+12	..	..
10	-34	-29	+3	-12	-10	-9	-4	+1	-31	+2	..	V. early.
11	-28	-22	-12	-14	+12	-11	-7	+10	-22	0	Late	Early?
12	(-29)	(-30)	+3	-3	-9	-6	-9	+2	-22	-8	Early?	Early.
13	+12	+18	+10	+11	+2	+3	-10	-9	+7	+11	..	..
14	..	..	-9	+14	+5	+10	-9	-5	+6	..	n.o.	..
15	(-19)	(-13)	-5	+4	-11	+10	-7	+7	-2	-11	Early	Early?
16	(+9)	(+13)	+12	-11	+12	+4	-3	+8	+22	-9	..	Late.
17	-17	-22	-1	-14	-8	-5	0	-5	-33	+11	Late?	Late?
18	-9	-9	-11	-4	+1	-11	+4	-10	-31	+22	Early	Late?
19	+14	+17	+7	+10	+6	-9	+7	+1	+22	-5	Late	V. late.
20	+40	+37	+7	+15	-11	-2	+9	+10	+28	+9	..	V. late.
21	(+26)	(+28)	-11	+5	+12	+7	+10	+2	+25	+3	Late	V. late.
22	(-14)	(-8)	-1	-9	-7	+11	+9	-9	-6	-2	n.o.	..

TABLE V.—*continued.*

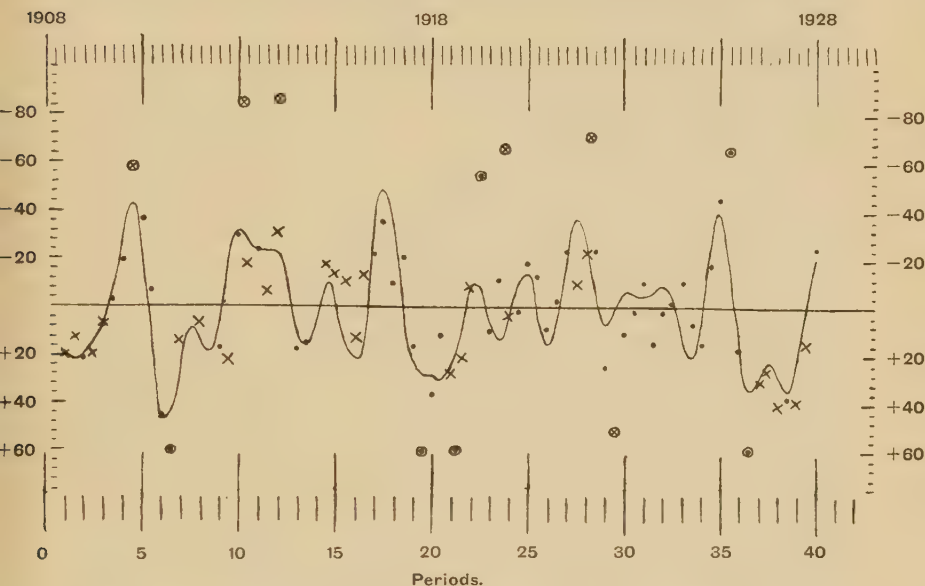
Period.	Observed Resids. (Portion 6).	Same corrected for "Season."	C ₁ .	C ₂ .	C ₃ .	C ₄ .	C ₅ .	C ₆ .	Sums of C's.	O-C.	Positions.	
											Min.	Max.
23	+ 9	+11	+12	-15	0	+ 8	+ 7	- 5	+ 7	+ 4	Late	V. late.
24	(- 3)	(+ 4)	- 6	- 6	+ 7	0	+ 3	+ 7	+ 5	- 1	n.o.	n.o.
25	-21	-18	- 9	+ 9	-12	- 9	0	+ 8	-13	- 5	..	..
26	+ 5	+10	+10	+15	+11	-11	- 4	- 5	+16	- 6	Late	..
27	-17	-23	+ 3	+ 7	- 7	- 6	- 7	-10	-20	- 3	..	Late.
28	(-26)	(-22)	-12	- 8	- 1	+ 3	- 9	+ 1	-26	+ 4	Early	V. early.
29	+31	+25	+ 4	-15	+ 8	+10	-10	+10	+ 7	+18	Late	n.o.
30	+ 7	+12	+10	- 7	-12	+10	- 9	+ 2	- 6	+18	..	..
31	-10	-10	- 9	+ 7	+10	+ 4	- 7	- 9	- 4	- 6	..	..
32	- 4	+ 2	- 5	+15	- 6	- 5	- 3	- 5	- 9	+11	Early	..
33	- 6	-10	+12	+ 8	- 2	-11	0	+ 7	+14	-24	..	Late.
34	+ 9	+16	- 2	- 7	+ 9	- 9	+ 4	+ 8	+ 3	+13	..	Late.
35	-42	-45	-11	-15	-12	- 2	+ 7	- 5	-38	- 7	..	Early.
36	+18	+18	+ 8	- 9	+10	+ 7	+ 9	-10	+15	+ 3	Late	V. early.
37	(+26)	(+31)	+ 7	+ 6	- 5	+11	+10	+ 1	+30	+ 1	..	Early.
38	(+45)	(+41)	-11	+15	- 3	+ 8	+ 9	+10	+28	+13	Late	Early ?
39	(+35)	(+40)	0	+ 9	+ 9	0	+ 7	+ 2	+27	+13	Late	Late ?
40	-28	-24	+12	- 5	-12	- 9	+ 3	- 9	-20	- 4	Early ?	Early.
Aver- age }	±20	±20	..	..	..	..	..	..	±18	± 8	..	..

The unit is one-hundredth of a magnitude.

In the diagram the intermediate points are also given, and the seasonal corrections have been applied with caution and moderation (see column 3 in the table). Brackets in the table and crosses in the diagram indicate that the position is insufficiently observed. In period 14, portion 6 was not observed at all, and this is the case also with portion 12 in several periods. These points are left without a dot or cross in the diagram. Crosses are also inserted for portion 9 in periods 10, 12, 21, and 28, and for portion 3 in period 24, these occupying outstanding positions. Portion 9 in period 6 would come lower down than portion 12 (which is given, ringed), and the same portion in period 35 would come higher up than the ringed dot given for portion 12. The "O-C" for the "portion 12" points is, of course, on the average, higher than for the "portion 6" points shown in Table V., the latter points being apparently very little affected by the phenomena shown in Table I. (Portion 6 comes into none of those examples.)

The first column in Table V. gives the number of the period of the main variation of R T Cygni; the second the observed mid-period residual magnitude; the third the same with "seasonal" correction where

that could be applied; the following six columns the calculated fluctuations: $C_1 = 12 \cos(\phi_1 - 160^\circ)$, ϕ_1 advancing by 106° per period of 191.2 days; $C_2 = 15 \cos(\phi_2 - 30^\circ)$, ϕ_2 advancing similarly by 59° ; $C_3 = 12 \cos(\phi_3 - 355^\circ)$, ϕ_3 advancing by 143° ; $C_4 = 11 \cos(\phi_4 - 292^\circ)$, ϕ_4 advancing by 48° ; $C_5 = 10 \cos(\phi_5 - 87\frac{1}{2}^\circ)$, ϕ_5 advancing by $22\frac{1}{2}^\circ$; $C_6 = 10 \cos(\phi_6 - 82^\circ)$, ϕ_6 advancing by 80° . The initial date for each of them is J.D. 2418037.6. The sum of these six is next given,



Observed and Calculated Secondary Variations in Magnitude of RT Cygni.

The full curve represents the calculated variations (C_1 to C_6), and the dots and crosses those found from the observations. Crosses indicate paucity of observations and rings drawn round dots or crosses call attention to the outstanding residuals, which have an appearance of abnormality. The figures at the foot refer to column 1 in Table V. The ringed points never rest upon less than 8 observations.

and then the difference (O-C) between column 3 and this sum. The final two columns state the early or late position of minimum and maximum, without attempting to give any precise date. (Dates have been obtained by analysis, but they are of no greater certainty than most of those given in the published papers.) A blank in these columns means that the date may very well be normal—viz. for minimum, 2418029.2 + n 191.2 days; and for maximum 2418120.5 + n 191.2 days ("n" equals one less than the number in column 1). The letters "n.o." mean "not observed." Each maximum and minimum is relatively bright or faint in accordance with the signs in columns 2 and 3.

Each of the six "C" fluctuations, taken by itself individually, reduces very considerably the sum of the residuals in column 3—the first

four waves do so by an average of over 110 each—and the reduction obtained by the sum shown in column 10 is not very much less than the sum of the six separate reductions. From the diagram it can be seen that, while these assumed fluctuations do not completely represent the observed variations, yet they approach nearly enough to doing so—*apart from certain outstanding residuals*—to suggest that they may approximate to the actually existing fluctuations. Still, even if this be so, it cannot be claimed that they give “the last word,” and they are sure to need some degree of modification in length, amplitude, or phase (or all three), when further data are available. There is also some evidence for several other possible waves—of (for example) 9-, 10-, or 12-period length. But the amplitude given by analysis is too small to be very trustworthy in each of these cases. C_5 and C_6 , too, are more doubtful than some of the earlier ones.

The markedly outstanding residuals are encircled by a ring in the diagram, and it will be seen that most of them present a scattered appearance, suggesting that they are not due to any smooth curve of variation, but are brief and sudden occurrences. The six C waves in almost all cases *reduce* them, but often by so small an amount that they seem to have but little connection with them. One of their chief features is the rapidity with which a rush in one direction is often followed by one in the opposite direction. Inadequate application of seasonal corrections could not account for such large variations as these. There are, however, certain circumstances which suggest some kind of periodicity. (1) Between the end of period 4 and the third quarter of period 6 there is a descent of well over a magnitude from a sudden abnormal brightness to abnormal faintness; and a similar phenomenon occurs between the third quarter of period 35 and the early part of period 37—nearly 31 periods later. About half-way between these two—from late in period 21 to the end of 22—exactly the reverse process occurs. (2) The early descent is followed before long by a rise rather similar to this central one, while the central rise is followed after nearly the same interval by a drop as great, sudden, and rapid as either of the others. The series does not continue long enough to show whether an abnormal sudden rise will follow the latest descent.

If there be any periodicity in these phenomena, it is apparently not of a regular wave character, but remains inoperative for long, and sometimes also for short, intervals of time.

A question arises as to whether the outstanding residuals, if really independent of the regular waves of fluctuation, may have unduly influenced the results of the analytical calculations for those fluctuations. (The first two C's are very nearly submultiples of $30\frac{1}{2}$ periods, the third of $30\frac{1}{4}$ periods, and the fourth, fifth, and sixth are exact submultiples of 30, 32, and $31\frac{1}{2}$ periods respectively.) But I do not think this can have occurred to any great extent, since the analysis was advisedly carried out from the mid-period figures, which (as already stated) appear to be very little affected by these phenomena. At the same time it is true that the mid-period figures used do not rest *solely* on the observations actually within portion 6 (often only one or two in number),

but these are combined with the means of portions preceding and following, giving, however, a greatly preponderating weight to the actual portion-6 observations. The intermediate points in the diagram were similarly obtained.

Suggestions might be offered as to possible explanations for both the regular and irregular variations; but at the present stage it seems better simply to present the observed phenomena and the tentatively assumed waves of fluctuation, without venturing on any hypothesis.

Summary.

1. A series of 1041 observations of R T Cygni, taken by A. N. Brown, extending over nearly twenty-one years, and comprising 40 periods, have been tabulated and discussed.

2. The mean length of period was found to be 191.2 days, and a smooth mean curve of variation was deduced, differing but little from a sine curve, but with the rise more rapid than the fall. The formulæ for the curve and for the mean dates found for maximum and minimum are given.

3. The residual magnitude, relatively to this curve, was found for every observation, and these residuals, being tabulated and examined, showed a number of very great and sudden changes in magnitude, at apparently irregular intervals (Table I.). Consideration of these led to the discovery of a well-marked half-yearly variation in apparent relative magnitude, continued fairly consistently throughout the series (Tables II. to IV.).

4. Apart from these sudden extremes of variation, an apparently systematic (though complicated) curve of fluctuation in relative magnitude was found and analysed, and a series of six calculated waves of different lengths was thus found. Their sum was compared with the observed fluctuations (Table V. and diagram).

5. These fluctuation waves, though they may approximate to the truth, will certainly require modification from further data, and other waves may also exist. Also, the period (191.2 days) found for R T Cygni's *main* cycle of variation, while it suits this whole series very well, is likely to need some correction eventually.

Observations of Novæ, 1928-1929. By W. H. Steavenson.

In the estimates of magnitude given in this paper the same comparison stars have been used as in former years. Unless otherwise stated, the observations were made with the 6-inch Wray refractor. When other instruments have been used, the estimates have been marked with an appropriate letter, as follows :—

L = 10-inch refractor of the University Observatory, Leiden.

G2 = 10-inch guider of the Greenwich astrographic equatorial.

Y2 = 12-inch refractor of the Yerkes Observatory.

H = 15-inch refractor of the Harvard College Observatory.

W = 26-inch refractor of the United States Naval Observatory,
Washington.

G = 28-inch refractor of the Royal Observatory, Greenwich.

Y = 40-inch refractor of the Yerkes Observatory.

I am much indebted to the respective directors of the above observatories for permission to use the telescopes named.

Nova Persei (1901).

The usual marked variability in the light of this nova has been maintained, the observed range amounting to 1^m.7 during the period covered by the observations. There is some suggestion of rhythm about the fluctuations between mid-August and mid-November, a period of the order of thirty days being indicated. The subsequent observations are not sufficiently numerous to show whether this rhythm was maintained beyond that time.

Date.	Mag.	Date.	Mag.
1928 July 11	12.32 L	1928 Sept. 21	12.91
12	12.48 L	22	12.79
Aug. 8	12.27 G	25	12.83
15	12.53	26	12.79
18	12.43	30	12.89
23	12.32	Oct. 1	12.91
25	12.32	3	13.03
27	12.32	4	13.18
28	12.32	5	13.18
29	12.22		13.13
Sept. 1	12.22	7	13.18
2	12.32 G	8	13.23
3	12.32	9	13.18
4	12.48	11	13.18
6	12.32	12	13.13
9	12.59	13	13.33
11	12.79	17	13.18
12	12.74 G	18	13.13
13	12.93	20	12.99
14	12.91	21	12.91 G
15	12.98 G	22	12.91
17	12.98	24	12.91
18	13.08	25	13.00
19	13.08	28	13.00 G
20	12.93 G	30	12.95

Date.	Mag.	Date.	Mag.
1928 Nov. 4	13.18 G ₂	1928 Dec. 29	13.13
8	13.03	31	13.18
15	13.18	1929 Jan. 12	13.03
16	13.33	Feb. 3	13.93
17	13.28	Mar. 1	13.87
19	13.18	4	13.87
23	13.42	10	13.87 G
Dec. 5	13.42	17	12.98 G
6	13.47	24	13.28 G
8	13.28	28	13.57 G
13	12.84	29	13.82 G
17	13.18	Apr. 1	13.47 G
21	12.89	7	13.37 G
22	12.91		

Nova Ophiuchi (1848).

The observations this year show scarcely any variability, whereas in past years the magnitude of the nova has often exhibited marked fluctuations. As seen under moderately good conditions with the 40-inch telescope on 1929 May 28, the image appeared to be sensibly stellar, and no nebulosity was seen in the vicinity.

Date.	Mag.	Date.	Mag.
1928 June 24	12.8 G	1929 May 10	12.8 H
July 1	12.7 G	16	12.8 W
29	12.7 G	17	12.75 W
Aug. 5	12.75 G	26	12.75 Y ₂
12	12.8 G	28	12.7 Y

Nova Aquilæ (1918).

There is very little variability in the residue of this nova, which has now practically ceased to fade. It is now close to the mean brightness which it exhibited before the outburst, but its visual magnitude is certainly not now subject to changes so great as those found on photographs taken before 1918 June. The image of the nova appeared stellar in the 28-inch telescope on August 12. The surrounding nebulous disc, recently photographed with the 100-inch reflector at Mt. Wilson, was too faint to be detected visually.

Date.	Mag.	Date.	Mag.
1928 June 1	10.8	1928 Aug. 5	10.8
July 7	10.9 L	12	10.8
11	10.7 L	18	10.8

Nova Cygni (1920).

This nova is still slowly fading. Seen with the 28-inch telescope on June 10 and October 21 it appeared stellar and free from nebulosity, but its focus was two or three millimetres longer than that of the 14th magnitude star which precedes it by about 25", in p.a. $285^{\circ} \pm$. The nova also appeared to be stellar in the 40-inch telescope on 1929 May 28.

Date.	Mag.
1928 June 3	14.5 G
10	14.65 G
July 11	14.7 L
Oct. 21	14.65 G
1929 May 28	14.8 Y

Nova Cygni (1876).

Marked variability was again noted in the light of this nova, which has exhibited similar fluctuations many times in the past. A drop of almost a whole magnitude in one week is shown by the two August observations. The image appeared quite stellar in the 40-inch telescope on 1929 May 28, and no nebulosity was seen round or near the nova.

Date.	Mag.	Date.	Mag.
1928 July 29	14.44 G	1928 Sept. 15	15.15 G
Aug. 5	14.48 G	20	15.1 G
12	15.4 G	Oct. 21	14.76 G
Sept. 2	15.00 G	1929 May 28	14.81 Y

Nova Lacertæ (1910).

The light of this nova is sensibly constant, and has been so for many years. The slight differences in the estimates as compared with those of a few years ago are probably attributable to the change from small to large apertures. The image of the nova appeared stellar and free from nebulosity both in the 28-inch and in the 40-inch telescopes.

Date.	Mag.
1928 Aug. 12	14.4 G
Sept. 15	14.4 G
20	14.3 G
1929 May 28	14.3 Y

Nova Aurigæ (1892).

Observed once only, on 1929 March 28, when it was estimated with the 28-inch telescope to be of magnitude 14.9. It has for some years now been of this same brightness.

Nova Geminorum (1912).

This nova was observed once only, on 1929 March 28. It was then estimated, with the 28-inch telescope, to be of magnitude 14.5. This observation, taken in conjunction with others made in recent years, confirms the existence of a slight degree of variability; but the general fading seems now definitely to have ceased. There was certainly a distinct nebulous glow around the nova, as often noted before; but, in view of other evidence, it seems quite possible that this is an optical effect, due to some peculiarity of the star's light in conjunction with the secondary spectrum of the large objective.

ERRATA.

Memoirs, 64, pt. 5, p. 168. $M = 90^\circ$, $e \cdot 58$; $\log \frac{r}{a}$, for 0.01777 read
0.10777.

„ „ „ p. 171. $M = 112\frac{1}{2}^\circ$, $e \cdot 79$; $\frac{v}{R}$, for 160 read 166.

Monthly Notices, 89, 509, last line, for $2\sigma_L = 0.61$ read $2\sigma_L^2 = 0.61$.

